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REALITY, IMAGERY AND FANTASY *

BY

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As I meditated upon the problem of addressing you as a man of science this evening, I went back in imagination a hundred and fifty years. I entered a craftsman's shop and looked around me. I saw the work bench, the planes, the chisels, and the simple devices of the craftsman of that age. I saw a primitive form of what today we would call a lathe, operated by foot, and I saw hand tools for the fashioning of round objects. The equipment was crude to the modern eye but served all the needs called for at the time.

And as I searched around, I came upon a cupboard which had apparently never been opened before by anyone but the craftsman who had built it. And I opened the cupboard that I might find such other objects as would give a clue to the future ambitions of the craftsman. To my astonishment, I found some electrical wire and several electrical switches. As I hunted further, I found several dynamos of complicated form. I found electric light bulbs, ready for use. Further research revealed an X-ray tube, and all the elements necessary to construct a wireless set, the elements being well fashioned to fit together in best form to realize their purpose. I found a television tube and the parts for constructing a telephone and, indeed, as I searched in the cupboard, I found more and more, till, when I had finished, I realized that there was no contrivance born of modern science for which I did not find in the cupboard of that ancient craftsman all the parts wherewithal to construct it. Truly, I was astonished in contemplating how all this had come to be. What manner of craftsman was this, who, while carrying on his peaceful trade of a century and a half ago, was

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yet making provision for all the richness which seemed to have no meaning or purpose in his day but which, in a new era, was to blossom forth in the fruits we now enjoy?

And with the picture of this craftsman and his strange shop, I would have you compare another—a very much older picture—a picture four or five billion years old, and one which concerns itself with a situation which is believed to have existed before the earth and the sun were born, before the stars were born, before any of the great galaxies of space were born and, indeed, before the atoms of matter itself were born. It is a picture which has been put together on the basis of studies of the old science of astronomy of our forefathers, of the newer science of cosmology, which is the astronomy of the universe, and of our more recent studies of atomic and nuclear physics. Many things have played a part in providing the possibilities of painting this picture. Our great telescopes have delved far into the depths of space, and have revealed to us innumerable objects which we believe to be copies, in different stages of evolution, of that little island in space with its sun, its stars, and its planets, that little island which constitutes our own local universe, and which, indeed, is only a part of the greater universe. These copies which are the brothers of our own universe in different stages of development, and which, indeed, are the great galaxies of space, have provided the means by which, through their study, we have sought to learn what we have been and what we may yet become. In some of these galaxies we see strange and awe-inspiring phenomena which dwarf into insignificance anything we have seen on our own earth or on our own sun. We have come to have considerable understanding of what is happening in these strange occurrences and this understanding has arisen from development, through experiments conducted for the most part on our own earth, experiments relating to the structure of atoms, of molecules, and all the other various entities which play a part in the activities of these things.

And so, returning to our picture, there is reason to believe that four or five billion years ago everything in the greater universe was collected together in one mass occupying no more space than is occupied by the orbit of Neptune; and that the stuff which then constituted all that existed was nothing but a conglomeration of neutrons and electrons, with a few protons. These particles were in a state of very high velocity. In fact, the situation was one of an enormously high temperature, a temperature of the order of a million times that of the sun's temperature today. Now a neutron is an unstable entity and, under ordinary conditions, dies by what we may call natural causes. As a matter of fact, the life expectancy of a neutron is only about 20 minutes. Its death is accompanied by the creation within itself of two charges, a positive proton and a negative electron. The negative electron is shot out, leaving the positive charge within the neutron, so

that the resulting entity becomes a proton. This process must be regarded as having been in operation during the extraordinary conditions of temperature to which I have referred. However, at such high temperature another process was also in evidence. On account of the high temperature, the electrons themselves flew about with very great velocity, so that, as they struck the protons which were among them, they were able to pierce an entry into those protons, annihilating their positive charges and converting them to neutrons. Thus, so long as the temperature conditions were maintained, there existed a certain balance between the number of neutrons thus created per unit of time and the number which died by the natural causes to which I have referred. However, if we imagine the temperature to have gradually decreased, a situation must have been reached in which the electrons which flew about in this inferno no longer had enough energy to enter the protons and re-create neutrons. Under these conditions, all of the neutrons would die very rapidly creating protons; and as the protons were produced, some of them would seek to combine with other neutrons; and by processes which are quite well understood at the present time, more complicated structures—the atoms of matter itself, to wit—must have started to be formed. The proton itself is a nucleus of the ordinary hydrogen atom. If a proton combines with a neutron, we have an atom called deuterium. If it joins partnership with two neutrons, we have an atom called tritium. If two protons and two neutrons somehow or other come into partnership, they form the atom which we call helium. And so we can understand how, by such processes, the various atoms of matter came into existence. Indeed, it is possible to trace the past history of this situation in terms of our present-day knowledge of the rates at which these processes would occur, and there results the astonishing conclusion that practically all the matter, practically all the atoms in the universe today, not only in our little universe—our little solar system, or our own galaxy—but in all the galaxies of space, were born in the limited space of half an hour. Now this whole picture may seem very fantastic to you, something which must have arisen in the minds of cranks. However, it is a matter which has been well considered by men of science, and I will venture to read a quotation from a man who has weighed the evidence of such men of science:

> It can be considered now as an unquestionable truth that "from one to ten thousand million years ago, the matter of the (known) spiral nebulae was compressed into a relatively restricted space, at the time the cosmic processes had their beginning" and that during that stage "the density, pressure, and temperature of matter must have reached absolutely enormous proportions" since "only under such conditions can we explain the formation of heavy nuclei and their relative frequency in the periodic system of elements."

Now you will naturally ask "Who is the author of this quotation?" The author is Pope Pius XII—and the reference is an address by Pope Pius XII to the Pontifical Academy of Sciences on November 22, 1951 (*Tipografia Poliglotta Vaticana*, Rome, 1951).

The atoms of the elements, or at least the nuclei of those atoms, the central cores, containing the neutrons and the protons, are very tightly bound structures, so that, until comparatively recently the resources of man were not such as to enable him to make any changes in those structures. It is only the extraordinary conditions prevailing in the bygone era of which I have spoken, conditions associated with enormous temperatures, which permitted changes in the nuclei of atoms, and the creation of new atoms, to take place spontaneously. Today, with our powerful electrical machines, we can, to some extent, compete with nature in this matter; but this is an achievement which is only about 20 years old. A certain amount of atom building is still going on under the high temperature conditions prevailing in the sun and the stars. In this particular kind of atom building, energy is released on an enormous scale. It is this supply of energy which keeps our sun from cooling. In its absence, the sun would cool very rapidly as the time scale of astronomy would interpret that phrase. The primary fuel for all such atom building is hydrogen, and some day, all the hydrogen will be used up.

Now the tightly bound structures which we call atoms can group themselves into less tightly bound structures called chemical compounds and of these the number is legion. The manipulations necessary to create such compounds is well under man's control. Indeed, the whole science of chemistry is concerned with putting together and breaking up the various groups of atoms which form chemical compounds. Some of these chemical compounds are extremely complicated. Instead of containing just a few atoms, as in the case of inorganic chemistry, some of them contain thousands of atoms, so as to result in structures which are on the point of detection by our most powerful microscopes. Some of these very complicated structures possess curious properties inherent in their architecture, properties which enable them to simulate to some extent the behavior of living things. Somehow or other, by processes which we do not yet completely understand but which are becoming gradually clearer and clearer, the phenomenon of life made its appearance in these complicated structures. The mystery of whether what we call life is simply an exemplification of the properties inherent in the complication, but an exemplification beyond our power yet to understand, or whether, superposed upon the properties inherent in the chemistry of the situation, there is something else, is a mystery which yet awaits solution. However this may be, what we call living things came into existence. And from simple structures there grew more complicated structures,

and then still more complicated structures beyond the limits of dreams, until animal life and plant life in their higher forms came into existence, and finally the development reached the stage of man himself, a creature imbued with such richness of potentiality that he dares to think back upon himself and try to solve the problem of his own existence.

And this creature, man, started to acquire experiences, and to devise means to meet them, means to fight his enemies, means to keep warm when it was cold, and to keep cool when it was hot, means to provide himself with further conveniences. And lo, he found in the as yet unused potentialities of that universe which was around him the very means for accomplishing these things which he so desired. The man of my earlier parable opens today the cupboard of the craftsman of 150 years ago and finds in there all the appliances necessary for the enrichment of his existence, but the man of my new story, which is no parable, finds throughout his history all the preparations for his growth in civilization, a preparation which was made five billion years before he came into existence, and in a period in which none of the things which he now enjoys could have existed for an instant in the great cauldron in which the universe was born. One of the strangest things to contemplate is the fact that all of the development of this richness, or of at least 90 per cent of it, has occurred within the last hundred years of man's existence on earth, a period so short that if the span of civilization was likened unto a day, that 100 years would represent only the last 20 minutes. And so, if I have to marvel at anything, I marvel at the extraordinary provision made by the craftsman of five billion years ago for the future needs of the products of his vision, needs which were not to be called upon until that distant era when life came into existence. If ever there was a plan, here indeed it is to be found.

When occasionally I hear science criticized as antagonistic to the concept of creation in casting doubt upon the literal interpretation of the story of Genesis, when I am asked to be content with the statement: "And God said, 'Let there be light,' and there was light," when I am asked this, I cannot but compare the rather transitory feeling of wonder invoked by the thought of God simply turning on a switch, with the awe inspired by the much grander and detailed story which science has unfolded as the mechanism behind the operations of the hypothetical switch. Man does not have to believe that he has understood the whole story in its totality to realize, from what he does understand, that there is here a magnificence beyond his dreams, a magnificence which can only grow in power with greater understanding.

And as man came into existence, he started to meditate upon the nature of his surroundings with a view to understanding them. As a rule, this understanding involved an attempt to see how the happenings which, at first sight, were to him incomprehensible, could be regarded

as particular cases of the happenings with which he felt himself familiar and which he therefore felt he did understand. In the earliest times, the things most evident to this primitive man were power to determine that things shall be done and strength to do them when strength was necessary. From the tribe of which any individual was a member there had emerged one man cleverer than the rest, a man who had acquired power. This man gave orders and things were done. And there had arisen men of great strength who could defeat others in battle. And so power and strength were the things which dominated the individual's thoughts. Primitive man did not concern himself much with the thought of where strength and power came from. That they existed in his own realm he knew as a fact, and he was prepared to grant their existence in other realms. And so things like the sun and the moon and the planets behaved as they did either because they themselves were things of power, power which enabled them to do what they pleased, or they were governed by other beings who had this power to decide the course of their actions. Thus, the sun and the stars and planets became gods or, if they themselves were not gods, then there were other invisible gods who dictated the course of their actions. And so, there was a period in which the behavior of things outside of the realm of ordinary daily life were to be understood in terms of power and the decrees of omnipotent beings. Understanding in terms of such power was understanding in the sense of *reality*, the reality of the era. Power was the tangible symbol of reality and if there was enough of it, anything could be done.

It is strange how long this reliance upon power as a basic starting point was regarded as satisfactory by mankind which in the age of the Greeks, for example, had attained such high stature in other things, in philosophy and more particularly in art. Indeed, there was quite an organization in this power, for there was a god for almost everything, so that when St. Paul visited Athens, he found a monument to the Unknown God, a monument erected evidently in fear lest there might be some god who was not properly worshipped and who yet was responsible for some important actions. And while the gods were in evidence as symbols of the reality which controls nature, there yet grew up a school of thought among philosophers as to the nature of the universe itself. This school of thought was formed, as it were, of the mind alone, without very serious control from the facts which nature presents. I hesitate to join too strongly in the customary criticism of it because that criticism frequently takes the form in which the later era governed by the works of Galileo, Newton and their contemporaries is claimed to represent an attack upon the problem based more solidly upon reality; and alas, as we shall see, the substance of such reality was, in turn, due to evaporate, so that once more man returned to creations of the mind in attempts to describe the pattern of nature.

However, these evolutions of the mind today are governed in large measure by nature as she presents herself to us rather than by preconceived notions born of the mind alone and supplemented by an intention somehow or other to force nature to be in accord with them.

The fact is that about the time of Copernicus, Galileo and Newton, philosophy took a new look at the phenomena presented to it and there evolved, first in astronomy, a wonderful scheme of correlations of the motions of the heavenly bodies in relatively simple form, a form so simple and powerful as to appear to claim a birthright as an exemplification of the ultimate fundamentals of the universe itself. At first Newtonian dynamics must have seemed very abstract to the scholars who lived when it was born. In those days reality was represented by omnipotent beings controlling the planets in their orbits and there must have been much straining of the imagination to understand how the spirit of the new formulation of the laws could be understood in terms of former concepts. For the new concepts seemed to invite strange difficulties. The sun seemed to have some kind of an action on the earth through space, and this naturally invited the cry "How can matter act where it is not?" Last year I showed how the people of those times might have been to some extent successful in moulding the new forms in such fashion as to make them reasonably understandable in terms of what might be expected as the natural properties of angels. I contended that even if complete success had been attained in this matter, the policy would have been a very bad one, because it would have postponed the adjustment of our mental processes to the Newtonian scheme of philosophy. While easing our philosophical difficulties for a time, the angels would soon have become an encumbrance, particularly in more complicated astronomical problems, where their insistence on being present would clash with more fundamental necessities.

The acceptance of the Newtonian philosophy had its good points and its bad ones. Its good points lay in the approach which a gradually increasing familiarity with the concept of forces, inertia and the like brought into existence to the end of securing a condition in which astronomical problems could be solved with ease and accuracy. However, the very development of this facility carried with it the conviction that the laws used for the solution were something to be regarded as almost obvious consequences of the fundamentals of logic. Truly there hung on the difficulty concerned with action at a distance and many were the attempts to fill intervening space between the planets with some kind of a medium which would serve the purpose of propagating light from one body to another and at the same time serve the purpose of transmitting force between the bodies in a way somewhat analogous to that in which force is transmitted from one end of a rod to the other when we push the said one end and find that, as a result,

the other end is able to move something to which it is attached. As a matter of fact, there can be much self-deception in seeking comfort in this continuity of action provided by a medium as something which may relieve us of the difficulties concerned with action at a distance; for all of our subsequent pictures of the mechanism of action in the medium seem to resolve themselves in the case of what we call material media, at any rate, to the operation of forces over small distances as compared with large distances, so that there is apt to be a hazy kind of contentment in the fact that if the distances are so small that we cannot imagine ourselves as seeing them, almost anything may be permitted to happen in those small distances without trouble to our conscience. However, many were more or less content to imagine that there might be something non-material in the ordinary sense, an all-pervading ether, which did not invoke particles to explain its action and which could serve as a perfect medium of continuity between the heavenly bodies and so relieve us of dependence upon that bogy, "action at a distance." It is questionable whether this attempt at escape from the supposed difficulty is more than the replacement of a dogma to the effect that "what the eye cannot be imagined to see, the mind need not trouble about" by another to the effect that "what the mind cannot understand is something which it need not trouble about." As a matter of fact, in the last analysis, the laws which control the behavior of a continuous non-material medium are just as abstract as are the laws of action at a distance, and we shall never succeed in alleviating our troubles in this matter until we come to realize that those troubles are things of our own creation, born of intuitions which, in turn, have no basis of fundamentality outside of the realm of our experiences in other things—things of a different nature. However, man sought to smooth over the remaining difficulties in these matters and to become more and more content in the idea of forces between various objects transmitted somehow or other by some kind of medium which provided continuity over all the universe. And so these things, ether, matter, forces, motion and laws of motion, became the substance of a new reality, so that one might expect to find himself content in the thought that all the revelations of nature could be understood in terms of such things. The great electromagnetic theory of Maxwell seemed to give some promise of an ultimate understanding in terms of such concepts. However, the new electromagnetic theory was difficult to understand, so that many had to be content with the promise itself, without attempting to comprehend its realization. Some of the more conservative minds felt that they were being cheated, in becoming beguiled into the acceptance of a new "reality" which, to them, was no reality at all, and rested strong in the faith that someday a reality acceptable to their own wishes would appear. The strength of conviction in this viewpoint is exemplified by a statement made by Sir

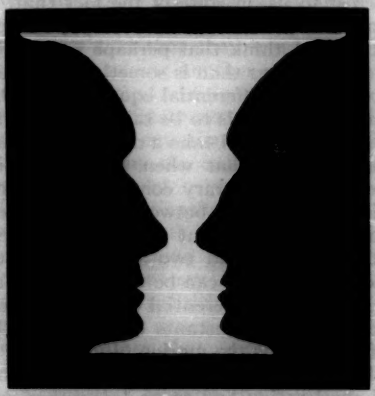
Arthur Schuster within the lifetime of many of us here this evening. Schuster was just one generation older than that to which I belong. Indeed, he was a man who lived and wrote in the period when I myself became active, a man with whom I have corresponded in my younger days. He was a great authority on optics, so that Maxwell's electromagnetic theory of light represented a philosophy which touched him deeply. Writing in his treatise on optics, and speaking of Maxwell's theory, he expresses himself as follows: "The study of physics must be based on a knowledge of mechanics, and the problem of light will only be solved when we have discovered the mechanical properties of the ether." Writing in another place of Maxwell's equations, he remarks: "The fact that this evasive school of philosophy has received some countenance from the writings of Heinrich Hertz renders it all the more necessary that it should be treated seriously and resisted strenuously." I have cited this quotation in earlier lectures before the Institute, but I feel the necessity of citing it again on the present occasion, because it exemplifies so concretely the spirit of the times with which it deals. And so, with this Newtonian-born reality there was an *imagery* which pictured nature as operating in a certain way which seemed natural; just as in the old days the images of things operative to control matters of the universe were angels, divine beings and the like, so now they were other things, media, something like water—the more like water the better—forces, something like the actions associated with our muscles but yet a little different, and so on.

And then, towards the end of the last century a whole new era was ushered in by the discovery of the electron, followed by the proton, and later by all of those so-called atomic particles with which we are becoming flooded more and more every day. And for a time the great electromagnetic theory of Maxwell, particularly in the hands of Lorentz, did yeoman service, and with considerable success, in correlating these new phenomena into the framework of that reality which was born of Newton's achievements, so much so, in fact, that most physicists of the turn of the century were willing to close their eyes to the lack of strict conformity with Newtonian mechanics, and to forego the imagery which it invited. One accepted the fact that he must not pin down reality too much to an accompaniment of imagery in the old sense, and the great Hertz, having tried hard to retain the imagery of the past, was driven to acknowledge failure and to assert that Maxwell's theory was Maxwell's system of mathematical equations and no more, a stand typical of those which brought forth Schuster's protest to which I have already referred. And so, in this epoch, physics became a mixture of images, and things which could not be pictured in the old sense. The dawn of wireless telegraphy cemented faith in the Maxwell theory, and that theory proceeded to see what it could do for the atom. The atom was to be a little solar system very much like that of our

own sun, and the electromagnetic theory of Maxwell permitted the electrons, which were the little planets of the atoms, to radiate energy in their course around their orbits. One hoped that this radiation of energy might explain the detailed nature of the behavior of light. However, the explanation was very imperfect and, unfortunately, the bad things about the prediction of this model were the very things which were insisted upon emphatically by the supposedly realistic characteristics behind it. The picture of the atom was a very beautiful thing, a very beautiful image for the mind to conjure with, but directly it was asked to do anything in line with the promises of the image, it fitted the facts, at best, badly and in actuality wrongly. It was like a beautiful opera singer who seemed fine until she opened her mouth and then she sang the wrong piece. And so it became necessary to do something about these discrepancies. It became necessary for us to suppose that the electron could do certain things that it was not supposed to do, like a human being suddenly relieved of the responsibilities of gravitation, so that he could fly off into space at will or could make himself disappear at one place and suddenly reappear at another. The fact is that the electron, the founder of the dynasty of modern physics, had acquired such prestige from the startling activities with which it had been associated at the time when the dynasty was founded, that it continued to claim credit for activities which were quite foreign to its supposed natural way of doing things. In the affairs of men in analogous situations, the founder of the firm would be retired. His picture would hang upon the wall while the business would avowedly be run along new lines. It might be that, out of past loyalty, it would result that in the case of any important action being taken, those concerned with the action would bow to the picture on the wall and proclaim that what was done was done in his name; but in actuality nothing would be expected of the old gentleman. Now while physicists are supposed to be logically minded people, their loyalty to the old gentleman—the electron—the monarch of the dynasty—is such that they continue to attribute all initiative and action to him. And so physics becomes like a conventional "king's speech," but given by a king who doesn't like the speech. To perpetuate the electron's authority in the light of the inconsistencies required of him, physicists seem perfectly willing to allow him to become a regular Dr. Jekyll and Mr. Hyde. Sometimes he is a particle, like a bullet, and sometimes he is a ghost or, if you will, a photon, behaving like a wave. And so the attempt to perpetuate imagery beyond the realm in which imagery can be consistently maintained leads science into the realm of *fantasy*, where strange things, such as are to be encountered only in dreams, control the activities of the new era. The facts of science have become the fairy tales of the old imagery and the old reality. And with the evaporation of the old concept of reality, there tended

to evaporate also the expectation of ever finding an understanding of nature founded upon such concepts. Philosophy became content to forego a quest for the "why" of things and concentrate on the "how." And so we tend to seek and be content in finding in nature a design rather than motivation for a design. If there is to be anything real in the matter, it is the design itself and the geometrical laws which govern its pattern. Nature is like an oriental rug in which our interest centers upon what we see rather than upon the question of who made the rug and why he made it. If I find that the whole complicated design can be made up of a number of simple structures, patterns, in some definite way, I am happy in the discovery that such is the case and grow a new kind of understanding of this rug. I may discover certain formations of patterns which occur again and again, in organized relationships, to produce in their totality other patterns which again may be repeated several times in such manner as to produce a still more richly organized pattern; and in the discovery of this richer pattern I am intrigued. In the very groupings of the pattern some will see significance in one place, others will see significance in another.

Here is a pattern. You will say: "It is a goblet." And if you



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exercise your mind in contemplation of this picture, the things which come to your vision are pictures of grand festivities, with this goblet and its fellows filled with bright sparkling wine. Let me now ask you to let your minds go blank for a moment and then look once more at the pattern, concentrating this time upon the black rather than upon the white. I think that now you see two men, apparently in conversa-

tion, and the meaning of the picture changes. Your thoughts change from the domain of feasts to those of discussions between able men. The over-all pattern is the same in both cases; but in your own mind you create different meanings for it, in accordance with the relative emphasis which you place on different features. And so in that great picture which represents the universe, different people may concentrate upon different features and one may maintain that the picture represents this, while another will maintain that it represents that. However, the picture is what it is and nothing more. The properties of the picture are properties of its form.

For a moment I should like to address myself to the professionally informed physicists in the audience, and for the special purpose of discussing further this matter of form in nature, as we see it, as distinct from what we usually refer to as the fundamental laws.

The so-called fundamental laws of physics usually take the form of differential equations. The aspects presented to us by nature are special solutions of these equations. While the special solutions are consistent with the fundamental laws, they have properties in their own right. Now it is frequently these special properties which are the things of interest to us, in spite of the fact that, from the fundamental standpoint, the custom has been to pay more homage to the differential equations. I think that perhaps we pay less attention to the properties of the *solutions* than is sometimes warranted.

If I take a very simple differential equation, $y = t$, the differential equation says everything that is to be said about the relation between y and t. On the other hand, if I take a differential equation dy/dt = a constant = a, then I know that when I have obtained the formal solution it will contain an arbitrary constant which must be assigned before the complete relationship between y and t is expressed. This constant expresses the value of y at $t = 0$. If I take a second order differential equation, I must add two arbitrary constants to the expression for the solution. They can be the value of y and the value of dy/dt at $t = 0$. If I have a differential equation of very high order, I must give a large number of constants to determine the solution. In fact, if the equation concerns the motion of a particle, I must give the initial position, the initial velocity, the initial acceleration, the initial rate of change of acceleration and so on. In other words, the higher the order of the differential equation, the more the information which I must add to that equation in order to tell the complete story. If the differential equation is of infinite order, I shall have to supplement it by giving the values of all the derivatives at $t = 0$. However, if I do this, Taylor's theorem tells me that I shall do the equivalent of specifying the complete relation between y and t, and no task is left for the differential equation.[2] In other words, the higher the order of the

[2] I am considering, for simplicity, an example where no singularities are involved.

differential equation, the less it says, and the more is left for me to say in order to express what I am talking about, until, in the limit, when the differential equation is of infinite order, it says nothing, and I have to say everything.

When I was a student studying the restricted theory of relativity, and learned that the laws of nature must be invariant under the laws of the Lorentzian transformation, I rather marvelled that it was possible to find laws which would satisfy this criterion. Indeed, unless one knows how to proceed about the matter, it is very difficult to find such laws, but if he knows the tricks of the trade, the difficulty is not great. However, when I came to the general theory of relativity and learned that according to that philosophy the laws of nature must be invariant under any transformation of coordinates, I began to wonder if it would ever be possible to find such laws. Yet, as we all know, there are such laws. However, when we examine the nature and content of these laws, we find that they say very much less than do the ordinary laws of prerelativistic physics. Thus, considering the celebrated equation of a geodesic, as it occurs in the general theory of relativity, we find that if we start to use it, we are confronted with certain complicated parts of it involving what are well known as the $g_{\mu\nu}$. If I ask for these in terms of the coordinates, so that I may proceed to a solution, I am told that they will not be given to me. I must find them myself by solving another set of differential equations in which I have to supply a lot of additional information which amounts in part to specifying the coordinate system in terms of which I intend to work. The fact is that the equations of the geodesic referred to say very much less than is said by the ordinary Newtonian equations of motion of a planet expressed, let us say, in polar coordinates in the usual fashion. If we say sufficiently little, we extend the scope to which our statements apply, so that if, in the last analysis, we say nothing at all, we say something sufficiently uncompromising as, indeed, to be invariant under any transformation of coordinates. Very general equations are like politicians—they have to accommodate themselves to so many different situations which, on the face of things, are different from one another that, compelled by the necessity of avoiding offence to anybody, they must talk in a language which does not say very much, however glowing the terms in which utterance is made.

Now the things that are important in nature are the patterns which it presents. The search for what we call fundamental laws is merely a search for the most economical statement of principles which permit all the patterns which are of interest to us. We seek a means in which, by saying few things, we may deduce as their permissible consequences many things, the many things expressing the totality of the richness of the universe. It is this richness, however, this pattern, which, as I

have said, is the thing of real importance. It is the expression of a purpose, if you will, and the more we look into the development of what is around us, the more do we find the concepts of form and pattern playing a dominant role. We see form in the structure of minute things, in the structure of the atoms. We see it in the still more minute structure of the nuclei of the atoms. We see it in the molecules and in innumerable compounds of atoms which nature affords. I would venture the suggestion that one of the great strides made, in our thinking, by the quantum theory is the recognition of the importance of pattern. Thus pattern is to be found in the various quantum states permitted for the atom. A mere differential equation normally says little about pattern, and it is perhaps only a matter of poetic justice that it should be a differential equation which, in the quantum theory, is given the task of deciding much concerning the pattern. The poor differential has to use a rather primitive device for performing this task so far removed from its normal field of activity. It has to rely on the assertion that if anybody puts into it any numbers (energies, for example) which would cause it to talk nonsense when it tries to talk at all—when it tries to deliver a solution—then those numbers are to be forbidden. The forbidding of such numbers, and the consequent limitation to other numbers which behave properly is equivalent to the assignment of a pattern in the relationship between different atomic states. All restriction rules in quantum theory, all parity laws and the like, are a further example of the recognition of pattern as distinct from the practices of the old easygoing era where everything which satisfied certain fundamental differential equations was permissible.

On a scale larger than that of molecular dimensions we see pattern continually in evidence. We see it in crystalline structure and we see it in the cellular structure which is fundamental to the organizations which we find in plant and animal life. We see it in the realization of that being man, himself, with his various organs fashioned to perform their individual functions, with his muscles and his limbs coordinated to an ideal of mechanical efficiency. Moreover, we see this pattern *developing* through the various stages of lower animal life as it gradually advanced to its goal in the human being. All the evidence presented by the universe around us today is an exemplification of ordered relationships anticipated in principle in that age long distant when there was naught but apparent chaos and confusion in what was the ancestor of our universe.

Such understanding as we have of what we call the physical world is a product of the last 300 years and most of it is confined to less than a century. In each stage of the organization of our minds toward an attempt to understand what is around us, we have sought that understanding in our experiences of the past, in the realities and pictures of

the days which have gone. With each new major discovery there was a period bordering on fantasy, a period in which the mind had to struggle to accommodate itself to new ideas until it could admit them into its own structure as things which once more assumed the garb of reality. In any era of advancement, our minds are apt to become so bound by the pictures of the past that a new field of discovery may seem not only fantastic but impossible. Only because discovery has come relatively slowly have we been able to accommodate ourselves at all to its implications. I wonder what a being of 150 years ago would have felt if he could suddenly have been transported from the realm of things then familiar to him, to a realm in which there was wireless telegraphy, television, airplanes, and the like. Most assuredly he would think that he had gone mad; and had he not *seen* these things, but had only been told of their existence by others who claimed to have seen them, he would most assuredly have claimed that his informers were strangely befuddled and had misinterpreted the evidence of their senses in what they had told him. Indeed, it would be interesting to contemplate the arrival of a modern airplane in the territory of an intelligent savage of our own time, but a savage who had never heard of our civilization. He would probably regard it as some kind of a supernatural bird; but if he told the story of its arrival to some of his fellows who had not seen the plane, he would probably be regarded as having gone crazy. In my own lifetime I have heard eminent men of science speak scoffingly of the belief in the existence of atoms which disintegrated spontaneously, men to whom the existence of radioactivity was something completely foreign to that fundamental belief in the permanence of atoms which their experiences of the past had forced into their thinking. Even to those who were working in the realm of radioactivity, the thought of *artificial* disintegration of the nuclei of the atoms was something which verged on the preposterous. I surmize that if those manifestations of atomic phenomena which have come to us in the last two or three decades had come, not by organized effort, but had come from isolated workers, not necessarily professional men of science, the world would have divided itself into two categories, the people characterized as crazy who said they had experienced these things, and those who, priding themselves on their common sense and rock-bottom saneness, would have asserted that such phenomena were beyond the realm of reason.

Again, if we should imagine that the elements of our science of today had possessed less immediate practical uses, and had been formulated as the result of investigations made by isolated people unorganized into groups of what we call scientists who, by communion one with another, could check one another's findings and formulate procedures by which meaningful results could be obtained; if, in fact, the nature of scientific discoveries had been such that no formal pro-

cedure for the harmonizations of discoveries had been formulated, then I think hardly any of the asserted discoveries of modern science would have been believed. I am quite certain that if such a group of unorganized people started to work on the ordinary elementary phenomena of what we call electrostatics, they would come to completely inharmonious conclusions. We know that these phenomena are highly sensitive to the proper conditions in the shape of a dry atmosphere, etc. If moisture is present there is electrical leakage, which tends to vitiate the main purpose of any particular experiment in all sorts of different ways. This kind of phenomenon would probably first make its appearance in such a form that those who encountered it would assert that the experiments would not work in the presence of water. Now, in times not very ancient, earth, fire, and water have played the part of rather mysterious concepts, and if some curious effect were said to be caused by the mere presence of water in the vicinity, the assertion would certainly be viewed with suspicion by all those who were sensitive to the evils of witchcraft. At best, he who made such assertions would be classed with the water diviners. Even when the effect of water vapor is more thoroughly understood, its elimination is not an easy process. I well remember the days in my youth when the trained professor of physics, having arranged his experiments in electrostatics for the benefit of his elementary class, was apt to feel that, on account of the insidious nature of this electrical leakage produced by the presence of water vapor in the atmosphere, it was more or less a matter of luck as to whether, when the time for the lecture came, the experiments would work.

If the cosmic radiation had been appreciably stronger than it is, I can imagine whole generations of people finding that they got sick to the point of death unless they lived in caves and yet they would have been completely without resources to detect the presence of these radiations in the sense in which we understand such detection today. Moreover, if somebody, after going into a uranium mine, or rather into a hole in the ground in which there was uranium, had recorded that he had become sick on account of his sojourn in this particular vicinity, there would be those who would attribute his ailment to the presence of a devil who resided there, but there would be many more who would say that everything was in his own imagination and that his sickness was a complete psychological illusion.

In a sense we human beings are simply entities like atoms and molecules, protons and neutrons, electrons, etc., in the whole realm of things which form the basic structure of the universe, and it is not without interest to contemplate how some of our brother entities with their known properties would view the situation around them if they could be imagined to be endowed with minds like our own. And here for a moment I must again address myself to the technical members

of the audience for the purpose of considering in some detail one or two rather simple illustrations.

It will be recalled that the electromagnetic equations are linear, so that if there be any one solution of these equations, we can superpose upon it any other solution and the sum of the two will be a solution. In other words, the presence of the second solution does not in any way interfere with the first. According to the views suggested by Sir Joseph Larmor many years ago, an entity like an electron is simply to be regarded as a singularity in one of these solutions, the place where the field becomes infinite in a certain way. Now if we should specify the field at each point of the universe at some instant, and if the field contained a singularity of this kind, then nothing more than the electromagnetic equations would be necessary to predict the future history of the field and along with it the motion of that singularity, which, on this view, is nothing more than a peculiar part of the field. If we consider another case of this kind, where there was one singularity in the field but the initial statement of the conditions was different from what it was before, then we should have another electron which moved about in a manner different from that of the former electron. If we now superposed these two solutions, the result would be a solution, and would represent a situation in which the two electrons moved in their own ways absolutely independently of each other. If there seemed to be some dependence, thus for example, if there seemed to be a tendency for the electrons to move away from each other, such dependences inviting the thought of repulsion, we should have to attribute the state of affairs to a mathematical accident inherent in the particular individual problems which we had chosen to produce the effects of the combination of two electrons. If somebody should specify for us the state of the universe at some instant as regards the electromagnetic fields, then all of the motions of the electrons would be determined for all time and again any appearance of relationship between them would be simply a kind of mathematical illusion born of the particular field system which had originally been specified. It is true that in practice we find it convenient to introduce what we call a force equation, which is said to control the motions of the electrons in relation to one another; but this force equation is simply something which is superposed upon what we had before, its function being that of making up for our lack of knowledge of the complete field specification concerned, a specification which, if known at some particular instant, would then determine the course of events for all time. In prinicple, these electrons moving about do not influence each other at all, the apparent relation between their motions being purely accidental in a mathematical sense. Now for one thing to be uninfluenced by another is to be unconscious of it. In other words, any consciousness of another's presence involves some kind of an influence, so we may say that, on this view of electromagnetic

theory, no electron in the universe would be aware of the presence of any other. The situation would be analogous to one in which every individual in the universe went his own sweet way, believing that he was the only creature in the universe. Some superbeing, looking at things as a whole, might see two such individuals going to a dance and performing gyrations in relation to each other; but in the absolute sense, any partnership in the dance would be an illusion, and each entity would be moving around the other while unconscious of the other's presence. So, in ordinary physics, and not only in modern physics, we have to admit sense to a scheme of philosophy which, if applied to human beings, would result in a situation more strange than any believer in the presence of invisible angels would ever have had to encounter.

Even in the most modern physics the conditions of one entity being completely unconscious of the presence of another is the normal condition, and only when we introduce what is called a perturbation does consciousness come into play. Frequently this consciousness is a very rare occurrence. Let me take an illustration. At the Bartol Foundation we are measuring the scattering of neutrons by matter. Now I should like you to consider what we should regard as a fairly sizable beam of neutrons, a beam containing about 10,000 million neutrons per square centimeter per second falling upon a piece of tin, one centimeter thick. As the neutrons go through the tin, some of them become scattered and it will be sufficient for us to regard each scattering as representing an encounter with an atom of the tin. That atom receives a kind of jolt as the result of the encounter. This jolt is an experience for that atom, a symbol of its consciousness of the presence of something else. It is interesting to inquire what fractions of the atoms receive such a jolt. We should find that of these 10,000 million neutrons passing through the tin per second, about 3000 million would be scattered. That is to say, about 3000 million atoms would experience a jolt each second, or about 10^{17} atoms would experience a jolt in a year. However, the piece of tin through which the neutrons passed would contain 600,000 times that number of atoms. Thus if one atom of the tin should claim to have received a jolt in a year, the claim would be less likely to be believed by the other atoms than would the claim of a human being that he had had a certain experience if in one year, he were the only person who had had such an experience in a city of 600,000 inhabitants. In practice, the situation is really much more startling; for in my illustration, I have confined my attention to the single cubic centimeter of tin which functioned in the experiment. If there were a cubic foot—that is, about 30,000 cubic centimeters—of tin in the laboratory as a whole, the fraction of all the tin atoms which had received the experience of a jolt would be reduced by a factor of 30,000, and the claim of any one atom to have had the experience

would be denied by 30,000 times as many of his partners as in my example.

Let us consider another example. Suppose that on our earth we had a thousand conical looking mountains, and that once in five million years one of these mountains produced a volcano. The chance that we should see a volcano in our lifetime would be extremely small; and if somebody claimed to have seen a volcano and reported such a thing in history, do you think that anybody would believe him? Yet, if you liken the blowing up of uranium atoms to the eruption of a volcano, the ratio of the blowing up of such atoms per five million years to the number present is just about equal to the ratio of the number of occurrences of a volcano on a conical mountain per five million years to the number of such mountains in my illustration.

And so I think we must be very cautious about denying the possibility of the occasional existence of phenomena which, in our day, may be called phenomena of an unusual nature. In a later era, such phenomena may become quite common. The spectacle of an automobile careening along the streets of Jerusalem at 80 miles an hour in the reign of King Solomon would have been regarded as an unusual phenomenon in those times, in spite of the fact that King Solomon had among his resources all of the wherewithal to make the tools which are necessary to make the parts which are necessary to make the components which are necessary to make the automobile. In fact, it would have been perfectly possible for King Solomon to have made the automobile if he had known how to go about it.

In the days of old the motions of the planets and, generally speaking, the motions of matter in bulk were the main things which presented themselves to our observation. Electrical phenomena were very rare. They were concerned with the attraction of small pieces of paper by a rod rubbed with the fur of a cat—they were matters of curiosity rather than of fundamental importance. Today in our civilization these electrical phenomena dominate everything. And so may it not be that we are headed for a future in which things that are met with only occasionally today, things which seem of little importance, will play a dominant role in the history of the future, a role even more significant than the role of mechanistic efficiency of our era?

And so in earlier stages of this address we have traced the supposed birth of the universe, its development along the lines prepared for it by that very extraordinary anticipation of potentialities, inherent in the laws of nuclear physics, which received their charter of operation five billion years ago. These laws dictated the organization of affairs in such fashion that in half an hour the basic structures of the universe, the atoms of matter, were formed. Then, over the remainder of the five billion years there developed the fruits of those potentialities which have given us the universe we know today. The story has been one of

development of what we may call the physical world. The development has been one of great magnificence, a development gigantic in its proportions, staggering in its boldness, and yet adorned at every stage with evidence of beauty, a beauty surpassing all creations of man! For who can paint the lily or redesign its form to better perfection? Man, the crowning achievement to date, has made his appearance and has been tempted to regard himself, even in his present form, as the ultimate goal of nature's achievements, the central figure of the universe, the crowning achievement of the Almighty, for whose service all that is in the heavens has been prepared.

And what of the future? I must be brief and cannot spare the time to provide reasons, but insofar as the spelling out of the genealogy of the past has claimed a power to foretell the future, we have some guidance in this matter. We see a situation in which, even at the present time, in our sun for example, the manufacture of elements is still going on. That manufacture is occurring at the expense of the ultimate fuel which is hydrogen. The manufacturing process provides the heat which the sun gives forth continually, and in the absence of this heat it would, as astronomy counts time, cool very quickly. Sooner or later this hydrogen will be used up, and the sun will start to cool, and consequently to contract in size. This contraction frequently leads to an instability of stellar bodies so that they explode into smaller fragments of very dense material which cool quickly. As a matter of fact, as we peer about in the universe, we find certain stars which have attained this degree of old age and it seems that in about ten or twelve billion years time we may expect to find nothing but the dead remnants of a glorious past, with the universe no more than a huge cemetery with no further life or potentiality of activity. The spectacle is a depressing one and one may well wonder whether any real intent of a purpose would have been achieved by it. Was the purpose really to create a fifteen billion-year spectacle? Was the universe made that it might be observed by what we call living beings during the brief lifetimes of such individual beings, or even during the brief lifetime of all life in the universe? If it is not this, are we merely confronted with an end which is naught but a confession of failure? Or has this titanic episode constituted simply a part of something else, something else which has not yet revealed itself, at any rate in its entirety, to those creatures, man and his kind, who have formed part of the material picture? Has this material universe constituted merely the chrysalis from which a very much more beautiful structure has been born, or is to be born, a structure in which the things to be spoken of are such things as souls, things which have left behind the battles of their birth, and formed a spiritual world where strife no longer is to be found, neither strife of human beings nor the strife of cosmical forces, and where death is no longer the inevitable end of the strife. If indeed

such is the case, we may join with the bard of Avon in likening all that we see around us to but a vision doomed to melt into thin air, a vision in which the cloud-capped towers, the gorgeous palaces, the solemn temples, the great globe itself, yea all that it inhabit, shall dissolve. But here, instead of joining with the great dramatist in the thought that not a rack will remain behind, we shall see something remaining, something which is indeed very real, something which was hidden from us in large measure during our mortal lives, but something which will remain as the only final reality after the material universe, which gave it birth, has died.

It is strange that man is so constituted that he frequently finds himself in a position in which, on Sundays, it is heresy to deny the things which it is heresy to accept on weekdays. And so, if the picture with which I have ended is fantasy, then make the most of it.

Tornado Tracking Service Considered.—The first step toward a tornado observing and warning network involving personnel from nine organizations in the state was taken at a meeting at The University of Michigan. Sponsored by the Southeastern Michigan Branch of the American Meteorological Society for the purpose of forming a "wider, more effective and more integrated" system for detecting, tracking and reporting tornadoes, the meeting resulted in the establishment of a small working committee to suggest a coordinated plan of action.

This committee is made up of representatives from the U. S. Weather Bureau, Michigan State Police and Office of Civil Defense under the chairmanship of state climatologist A. H. Eichmeier of Lansing.

As envisioned, the network would involve personnel from those groups, "ham" radio operators, University of Michigan radar experts and meteorologists, the American Red Cross and the Michigan Sheriff's Association, and would cover the lower half of the state's lower peninsula, with "spotters" every few miles. At the same time, radar sets such as those at the U-M's Willow Run Laboratories or planned for the Wayne Major Airport could be on the alert for severe storms. Information from ground and radar observers would be relayed to the Weather Bureau and the public notified in the areas involved. The need for speedy and thorough communications between the groups concerned was emphasized during the session.

Observers must also be trained to identify tornadoes positively, so that false alarms are not sounded or tornadoes ignored which lack the characteristic funnel-shaped clouds, the conference heard.

University of Michigan meteorologists told of picking up severe storms by radar as they moved inland from Lake Michigan during the last spring's series of tornadoes, and of tracking them across the state and into Canada and Pennsylvania.

Improved Additives for Lubricating Oil.—A new series of automotive lubricating oil additives which make engine starting easier in cold weather and also improve high temperature performance has been announced today by the Industrial Chemicals Division of Olin Mathieson Chemical Corporation. Developed in the company's research laboratories at New Haven, Conn., the new materials are classified chemically as oil-soluble, high - molecular - weight polymers. They are of petroleum origin. The additives will be marketed to lubricating oil manufacturers under the name Omavis.

In automotive crank case oils, they decrease the tendency of the oil to thin out in the hot spots of the engine, thereby reducing mechanical wear. Also, by lowering the congealing point, they increase flowability of the oil at sub-zero temperatures. Tests conducted in the company's laboratories show that oil containing Omavis flows readily at temperatures of -30 to $-50°$ F.

"The trend in the automotive field toward engines with more power and higher compression ratios means that oils today must be built for more rugged service than was the case ten years ago," according to Donald F. Ryder of the Industrial Chemicals Division market development staff.

"Top quality oils for automotive use are now fortified with several times the dosage of chemical additives required prior to 1945. The use of additives in oil will continue to grow as performance requirements become even more exacting," he said.

NUMERICAL INTEGRATION OF $y'' = \phi(x,y,y')$ USING OSCULATORY INTERPOLATION *

BY

HERBERT E. SALZER [1]

ABSTRACT

This present paper presents several new procedures, with formulas, for the stepwise integration of $y'' = \phi(x,y,y')$, starting from some initial values of y, y' and y''. These methods are based upon different types of osculatory interpolation formulas. One method uses an approximation to the next y'' by a formula for osculatory extrapolation for the derivative, and then finds the next y' and y using formulas for $n\frac{1}{2}$-point osculatory interpolation and $(n+1)$-point osculatory quadrature respectively. Another method is applied to the special case of $y'' = \phi(x,y)$, employing some new interpolation formulas based upon the function and its second derivative. Finally, for $y'' = \phi(x,y,y')$ the most accurate set of formulas utilizes "hyperosculatory" interpolation, which is based upon the function with its first and second derivatives. Predicting formulas, which are purely extrapolatory, give the next y and y' (then y''). Refining formulas employ the next y'' in "$n\frac{1}{2}$-point" hyperosculatory formulas to find a new y', and then they employ y'' and this new y' in "$n\frac{1}{2}$-point" hyperosculatory formulas to find a new y (then a new y''). This refining process may be repeated if necessary.

1. INTRODUCTION

In a previous paper (1)[2] the author discussed the numerical integration of $y' = \phi(x,y)$ employing highly accurate new formulas based upon osculatory extrapolation or interpolation. The purpose of this present article is to present similar new procedures for integrating $y'' = \phi(x,y,y')$, based upon several different types of osculatory interpolation formulas. Retaining the same notation as in the earlier paper, we assume that we have a certain number n of initial values of y, y' and y'', say y_i, y_i', and y_i'', $i = -[(n-1)/2]$ to $[n/2]$** and wish to find the next values of y, y' and y'', say $y_{[n/2]+1}$.

2. OSCULATORY FORMULAS FOR $y'' = \phi(x,y,y')$

One method which is highly accurate is to estimate the next value of y'' by some kind of extrapolation, then to obtain the next y' using the author's newly-found $n\frac{1}{2}$-point osculatory interpolation formulas (1) for obtaining $f_{[n/2]+1}$ from f_i, $i = -[(n-1)/2]$ to $[n/2]$ and f_i', $i = -$

* The present version of this article, which gives remainder terms for almost all formulas, is a revision of an earlier one (without remainder terms for most formulas) that was completed while the author was at the Diamond Ordnance Fuze Laboratories.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

** In this present paper $[y]$ will denote the largest integer not exceeding y only when y is $(n-1)/2$ or $n/2$; otherwise it is a parenthesis notation.

$[(n-1)/2]$ to $[n/2]+1$, where here $f \equiv y'$ and $f' \equiv y''$, and finally to calculate the next y with the aid of the author's recently published osculatory quadrature formulas (2) for obtaining $\int_{[r/2]-1}^{[r/2]} f(x)dx$ from f_i and f_i', $i = -[(r-1)/2]$ to $[r/2]$, where also here $f \equiv y'$ and $f' \equiv y''$. This process may be iterated in its second two stages, namely, the computation of the next y' and y, but now starting with a more accurate $y''_{[n/2]+1}$ obtained from the equation $y'' = \phi(x,y,y')$ instead of extrapolation, until both $y'_{[n/2]+1}$ and $y_{[n/2]+1}$ are of sufficient accuracy to proceed further.

The stability of this procedure with respect to the propagation of error due to the size of the coefficients in the formulas can be roughly surmised from the following consideration: Suppose that the extrapolated (next) y'' has error around E. Then the $n\frac{1}{2}$-point formulas (1) will multiply that term by around $h/3$ (h is the tabular interval), to give around $Eh/3$ for the error in y'. The size of the coefficients in the osculatory quadrature formulas (2) shows that errors in the next y'' and y' contribute an error to the next y of about

$$\left(\frac{h}{3}\right)(\text{error in } y') + \frac{h^2}{10}(\text{error in } y'') \quad \text{or} \quad \frac{h^2}{9}E + \frac{h^2}{10}E,$$

the total being about $(h^2/5)E$.

The reader may raise the question as to whether the above error estimate of $(h^2/5)E$ is better than that obtained by either using twice in succession a single quadrature formula for obtaining y' from y'' and then y from y', or by using a single quadrature formula for obtaining y' from y'' followed by a double quadrature formula for obtaining y from y''. (In connection with the latter process the reader is referred to several articles which contain formulas for double quadrature, both for use without differences (3) and for use with either advancing (4) or central (5) differences.) The point is that even though the magnitudes of the coefficients in the single and double quadrature formulas are such as to indicate a net total error which is comparable in amount, namely, $(Eh^2)\times$ (fraction of same order as 1/5), that is only the error due to inexactitude in the initial values. The really profound advantage in the present new procedure over the iterated single or single-and-double quadrature procedure, is that E is much smaller; all formulas for the next y' and y (and even the next y'' according to the paragraph below) are based upon osculatory interpolation or integration (differentiation for y'') so that they are essentially of the order of accuracy of $(2n)$-point formulas. This means that we can use a much larger value of h with a great saving in the number of steps and total computational work.

We do not rely upon ordinary extrapolation to find $y''_{[n/2]+1}$, but use a new type of *osculatory extrapolating numerical differentiation* formula

which gives $f'_{[n/2]+1}$ in terms of f_i and f_i', i ranging from $-[(n-1)/2]$ to $[n/2]$. Thus instead of basing the next y'' upon the n preceding values of y'' only, we base it on the n preceding values of both y'' and y'. The resulting formula has $(2n-1)^{\text{th}}$ degree accuracy. (That is, it is exact for polynomials of degree not exceeding $(2n-1)$, which corresponds to the accuracy of the other formulas we shall use. The new extrapolating numerical differentiation formulas are obtained by differentiating the corresponding Hermite osculatory interpolation polynomials (for which see (9), pp. 85–88).

In the schedule below, under the heading of "n-point," we give first the n-point formula for the next y'', followed by the $n\frac{1}{2}$-point formula for the next y', and then the $(n+1)$-point formula for the next y.[3] The entire 1-point schedule and the 2- and $2\frac{1}{2}$-point parts of the 2-point schedule are well-known formulas. The formulas for $y''_{[n/2]+1}$, beyond the 2-point schedule, are new; those for $y'_{[n/2]+1}$ are taken from (1) without modification in form, and those for $y_{[n/2]+1}$ are taken from (2) with slight modification in form to facilitate machine calculation. This slight repetition of subject matter makes this paper self-contained.

The remainder terms for $y''_{[n/2]+1}$ were calculated by the methods in (6), pp. 64–68, ex. 8 p. 86), the remainder terms for $y'_{[n/2]+1}$ are given essentially in (7) (also an equivalent method for computing them was communicated to the author privately by T. N. E. Greville), and the remainder terms for $y_{[n/2]+1}$ are taken from the author's earlier paper (2, p. 112).

3. SCHEDULES FOR NUMERICAL INTEGRATION OF $y'' = \phi(x,y,y')$

1-Point Schedule

$$y_1'' = y_0'', \text{ error} = hy^{(3)}(\xi),$$

$$y_1' = y_0' + \frac{h}{2}(y_0'' + y_1''), \text{ trapezoidal rule, error} = -\frac{h^3}{12}y^{(4)}(\xi),$$

$$y_1 = y_0 + \frac{h}{2}(y_0' + y_1') + \frac{h^2}{12}(y_0'' - y_1''), \text{ error} = \frac{1}{720}h^5y^{(5)}(\xi), \quad x_0 \le \xi \le x_1.$$

2-Point Schedule

$$y_2'' = \frac{12}{h}(y_0' - y_1') + 5y_0'' + 8y_1'', \text{ error}^4 = \frac{h^3y^{(5)}(\xi)}{2},$$

$$y_2' = y_0' + \frac{h}{3}(y_0'' + 4y_1'' + y_2''), \text{ Simpson's rule, error}^4 = -\frac{h^5y^{(6)}(\xi)}{90},$$

[3] Thus under the n-point heading we have three formulas of ascending accuracy: for next y'', based on y_i' (n points), y_i'' (n points); for next y', based on y_i' (n points), y_i'' ($n+1$ points); for next y, based on y_i' ($n+1$ points), y_i'' ($n+1$ points). The respective accuracies (order of the approximating polynomial involved) are $2n-1$, $2n$, $2n+1$ in y', or $2n$, $2n+1$, $2n+2$ in y. Note, however, that the formulas for the next y, y' involve inexact values of the next y', y''.

[4] See (8, p. 48).

$$y_1 = y_1 + \frac{1}{240}\{h(11y_0' + 128y_1' + 101y_2') + h^2(3y_0'' + 40y_1'' - 13y_2'')\},$$

$$\text{error} = \frac{1}{9450}h^7y^{(7)}(\xi),$$

$$x_0 \leq \xi \leq x_2.$$

3-Point Schedule

$$y_1'' = \frac{3}{h}(11y_{-1}' + 8y_0' - 19y_1') + 10y_{-1}'' + 57y_0'' + 24y_1'', \text{ error} = \frac{11}{60}h^3y^{(7)}(\xi),$$

$$y_2' = y_{-1}' + \frac{27}{11}(y_0' - y_1') + \frac{3h}{11}\{(y_{-1}'' + y_1'') + 9(y_0'' + y_1'')\}, \text{ error} = -\frac{3}{1540}h^7y^{(8)}(\xi),$$

$$y_2 = y_1 + \frac{h}{18144}(397y_{-1}' + 2403y_0' + 8451y_1' + 6893y_2') + \frac{h^2}{30240}(163y_{-1}''$$

$$+ 2421y_0'' + 7659y_1'' - 1283y_2''), \text{ error} = \frac{313}{254\,01600}h^9y^{(9)}(\xi),$$

$$x_{-1} \leq \xi \leq x_2.$$

4-Point Schedule

$$y_1'' = \frac{10}{h}\left(\frac{55}{9}y_{-1}' + 24y_0' - 15y_1' - \frac{136}{9}y_2'\right) + \frac{47}{3}y_{-1}'' + 184y_0''$$

$$+ 264y_1'' + \frac{152}{3}y_2'', \text{ error} = \frac{5}{84}h^7y^{(9)}(\xi),$$

$$y_1' = y_{-1}' + \frac{32}{5}(y_0' - y_1') + \frac{6h}{25}\{(y_{-1}'' + y_1'') + 16(y_0'' + y_1'') + 36y_1''\},$$

$$\text{error} = -\frac{h^9y^{(10)}(\xi)}{2625},$$

$$y_2 = y_2 + \frac{h}{43\,54560}(59681\,y_{-1}' + 6\,13456y_0' + 7\,11936y_1' + 14\,29936y_2' + 15\,39551y_3'$$

$$+ \frac{h^2}{7\,25760}(2237y_{-1}'' + 49720y_0'' + 1\,83708y_1'' + 2\,49656y_2'' - 26051y_3''),$$

$$\text{error} = \frac{551}{3143\,44800}h^{11}y^{(11)}(\xi),$$

$$x_{-1} \leq \xi \leq x_3.$$

5-Point Schedule

$$y_2'' = \frac{10}{9h}\left(\frac{685}{8}y_{-2}' + 739y_{-1}' + 351y_0' - 899y_1' - \frac{2213}{8}y_2'\right) + \frac{131}{6}y_{-2}''$$

$$+ \frac{1295}{3}y_{-1}'' + 1270y_0'' + \frac{2440}{3}y_1'' + \frac{535}{6}y_2'', \text{ error} = \frac{137}{7560}h^9y^{(11)}(\xi),$$

$$y_2' = y_{-2}' + \frac{1}{137}[1625(y_{-1}' - y_2') + 2000(y_0' - y_1') + 30h\{y_{-2}'' + y_2''$$

$$+ 25(y_{-1}'' + y_2'') + 100(y_0'' + y_1'')\}], \text{ error} = -\frac{5}{63294}h^{11}y^{(12)}(\xi),$$

$$y_2 = y_2 + \frac{h}{228\,09600}(2\,21349y_{-2}' + 36\,16235y_{-1}' + 86\,48560y_0' - 3\,62640y_1'$$

$$+ 30\,53035y_2' + 76\,33061y_3') + \frac{h^2}{53\,22240}(10795y_{-2}'' + 3\,45053y_{-1}'' + 19\,14300y_0''$$

$$+ 31\,31972y_1'' + 23\,35779y_2'' - 1\,68491y_3''), \text{ error} = \frac{80\,67077}{2876\,88360\,96000}h^{13}y^{(13)}(\xi),$$

$$x_{-2} \leq \xi \leq x_3.$$

4. SPECIAL FORMULAS FOR $y'' = \phi(x,y)$

When in $y'' = \phi(x,y,y')$ either y or y' or both y and y' are absent, the integration procedure may be simplified. Thus in the last case we have merely a double quadrature to perform upon $y'' = \phi(x)$, formulas for which have already been mentioned above (3–5). When y is missing, $y'' = \phi(x,y')$ is only a first order equation, in which case one of the most accurate procedures for integration is to determine y' by the formulas in (1) and then y by the formulas in (2). The case where y' is missing is still capable of treatment by the above methods which utilize y'', y' and y as long as there are initial values of y'. But owing to the great importance and frequent occurrence of the special case $y'' = \phi(x,y)$, it is worth investigating the existence of shorter and more direct procedures for obtaining the next y from tabulated values of y and y'', which would be analogous to the methods in (1) based upon y and y'.

An investigation was made of n-point and $n\frac{1}{2}$-point formulas of the type

$$(A) \qquad y_{[n/2]+1} = \sum_{i=-[(n-1)/2]}^{[n/2]} (A_i y_i + h^2 B_i y_i'')$$

and

$$(B) \qquad y_{[n/2]+1} = \sum_{i=-[(n-1)/2]}^{[n/2]} C_i y_i + h^2 \sum_{i=-[(n-1)/2]}^{[n/2]+1} D_i y_i''$$

respectively, for the first few values of n, with the following rather interesting results:

For $n = 1$, there is no formula of type (A) or type (B), for general $y(x)$. For type (A) requires a first degree polynomial for which y_0, $y_0''(\neq 0)$ are specified; type (B) requires a polynomial of second degree for which y_0, y_0'', $y_1''(\neq y_0'')$ are specified.

For $n = 2$, we have (A) $y_2 = -y_0 + 2y_1 + h^2 y_1''$ and (B) $y_2 = -y_0 + 2y_1 + \frac{h^2}{12}(y_0'' + 10y_1'' + y_2'')$, with respective errors $\frac{h^4 y^{(4)}(\xi)}{12}$ and $-\frac{h^6 y^{(6)}(\xi)}{240}$, $x_0 \leq \xi \leq x_2$. Neither of these expressions is new, the former being the very well-known $\delta^2 y_1 = h^2 y_1''$ and the latter another expression of the widely used formula $\delta^2 y_1 = h^2(y_1'' + \frac{1}{12}\delta^2 y_1'')$ (see (8), p. 84).

For $n = 3$, it can be proven that no formula of type (A) exists,[5] but there is a $3\frac{1}{2}$-point formula of type (B), of sixth degree accuracy in y, namely, $y_2 = y_{-1} - 3y_0 + 3y_1 + h^2\left\{\frac{1}{12}(y_2'' - y_{-1}'') + \frac{3}{4}(y_1'' - y_0'')\right\}$,

[5] The assumption of the existence of such a formula leads to an incompatible system of linear equations, as for $n = 1$.

with *dominant* error term $-\frac{h^7 y^{(7)}(\xi)}{240}$, $x_{-2} \leq \xi \leq x_2$. (The author has not yet found the *exact* expression for the error; it *might* be just that.) However that formula can be shown to be not new, since it follows by subtracting

$$\delta^2 y_0 = h^2(y_0'' + \frac{1}{12}\delta^2 y_0'' - \frac{1}{240}\delta^4 y_0'' + \cdots)$$

from

$$\delta^2 y_1 = h^2(y_1'' + \frac{1}{12}\delta^2 y_1'' - \frac{1}{240}\delta^4 y_1'' + \cdots),$$

giving an exact answer when y'' is of the fourth degree or less, since the $\delta^4 y''$ terms are constant and drop out. At any rate, this $3\frac{1}{2}$-point formula can be used by itself, to a limited extent, first as a predictor with an extrapolated y_2'' and then as its own corrector employing an improved y_2'' obtained from $y_2'' = \phi(x_2, y_2)$.

For $n = 4$, we have the first n for which there are formulas of types (A) and (B) and which are apparently new (at any rate, not readily derivable from other known formulas). The 4-point purely extrapolatory formula of seventh degree accuracy is the following:

$$y_3 = -y_{-1} - 16y_0 + 34y_1 - 16y_2 + \frac{4h^2}{3}(2y_0'' + 11y_1'' + 2y_2''),$$

and the corresponding $4\frac{1}{2}$-point refining formula, which has the surprisingly pleasant feature of affording ninth degree accuracy in y instead of the anticipated eighth degree accuracy,[6] is given by

$$y_3 = -y_{-1} + \frac{1}{31}[318y_1 - 128(y_0 + y_2)] + \frac{h^2}{465}[23(y_{-1}'' + y_3'') + 688(y_0'' + y_2'') + 2358y_1'']$$

The errors in these formulas are $O(h^8)$ and $O(h^{10})$, respectively. The author has not yet worked out the coefficients of h^8, h^{10} in the leading terms of these errors.

For $n = 5$, as for the preceding odd values of n, one can again prove that no formula of type (A) exists. This fact, together with the mounting size of the coefficients, the sufficiency of what is already available through the 4-point and $4\frac{1}{2}$-point pair, and the unexpected accuracy of the $4\frac{1}{2}$-point formula, all acted to discourage the further pursuit of formulas of types (A) and (B).

[6] We are not justified in expecting every such $n\frac{1}{2}$-point formula to have $(2n + 1)$th degree accuracy in y because even though that happens to be the case for $n = 2$ and 4, it is not the case for $n = 3$ which has the expected sixth degree accuracy.

5. HYPEROSCULATORY FORMULAS FOR $y'' = \phi(x,y,y')$

In this present section we return to the equation $y'' = \phi(x,y,y')$ with all the variables there. We develop a procedure that is more characteristic of the second order in that it utilizes "hyperosculatory" interpolation[7] where one bases the interpolating polynomial upon the function and its first and second derivatives at n points. The preceding formulas were of approximately $(2n)^{\text{th}}$ degree accuracy in y; the formulas in the schedules following this present section are of approximately $(3n)^{\text{th}}$ degree accuracy.

The general procedure is to start with a certain number of initial values of y_i, y_i' and y_i'', at n points, the i ranging from $-[(n-1)/2]$ to $[n/2]$. First we obtain the next y_i or $y_{[n/2]+1}$ by hyperosculatory extrapolation which is based upon those $3n$ values of y_i, y_i' and y_i'', the formula being of $(3n-1)^{\text{th}}$ degree accuracy. Then the next y_i' or $y'_{[n/2]+1}$ is obtained using a formula for *hyperosculatory extrapolating numerical differentiation* based upon those same initial $3n$ values of y_i, y_i' and y_i''. Finally, $y''_{[n/2]+1}$ is found from $y'' = \phi(x,y,y')$. This yields the next set of y_i, y_i' and y_i'' which are now included in some hyperosculatory refining formulas that are used in the following order. First we obtain $y'_{[n/2]+1}$ from the originally given $3n$ values of y_i, y_i' and y_i'' together with $y''_{[n/2]+1}$. Such a $(3n+1)$-condition formula will be called "$n\frac{1}{3}$-point hyperosculatory." Then we obtain $y_{[n/2]+1}$ again, but this time basing the refining formula upon the $y''_{[n/2]+1}$ and upon the $y'_{[n/2]+1}$ that was just obtained by the $n\frac{1}{3}$-point formula, as well as upon the initial $3n$ values of y_i, y_i' and y_i''. This last type of $(3n+2)$-condition formula will be called "$n\frac{2}{3}$-point hyperosculatory." Finally, a still better $y''_{[n/2]+1}$ is obtained from $y'' = \phi(x,y,y')$. This refining process may be repeated if necessary. The refining formulas, besides being more accurate than the predicting formulas because they are based upon one or two more values of y' or y'', will also tend to depress the error introduced into $y_{[n/2]+1}$, $y'_{[n/2]+1}$ (and indirectly into $y''_{[n/2]+1}$) by the size of the coefficients in the predicting formulas, since the multipliers of $y''_{[n/2]+1}$ and $y'_{[n/2]+1}$ are small.

The n-point schedules of predicting and refining formulas given below were not obtained for $n > 3$, because when $n = 3$ the $3\frac{2}{3}$-point formula is already a tenth degree approximation for y, all within a total range of $3h$. The 1-point schedule contains no new formulas, being included for completeness.

The remainder terms for the 2- and 3-point formulas were obtained as follows: for predicting formulas for $y_{[n/2]+1}$, from the general expression in (9), p. 86; for predicting formulas for $y'_{[n/2]+1}$, and refining formulas for $y_{[n/2]+1}$, by the methods in (6), pp. 64–68, ex. 8, p. 86, applied to the

[7] The term "hyperosculatory" is used here to emphasize the presence of the second derivative, but the term "osculatory" is also used by many authors to cover interpolation with any number of successive derivatives.

general Hermite osculatory interpolation formula. One derives refining formulas for $y'_{[n/2]+1}$, by a slight trick; first one obtains y'' for arbitrary x by the double differentiation of Hermite's formula for $y(x)$ in terms of fixed y_i, y_i', y_i'', $i = -[(n-1)/2]$ to $[n/2]$, and $y_{[n/2]+1}$; then (going one stage beyond Greville's technique) one sets the parameter $x_{[n/2]+1}$ equal to λ and the variable x equal to $x_{[n/2]+1}$ *in that order*; then one differentiates again with respect to the new variable λ, and finally sets λ equal to $x_{[n/2]+1}$.

6. SCHEDULES OF HYPEROSCULATORY FORMULAS FOR INTEGRATION OF $y'' = \phi(x,y,y')$

1-Point Schedule

Predicting:

$$\left.\begin{aligned} y_1' &= y_0' - hy_0'', \text{ error} = \frac{h^2}{2}y^{(3)}(\xi), \\ y_1 &= y_0 + hy_0' + \frac{h^2}{2}y_0'', \text{ error} = \frac{h^3}{6}y^{(3)}(\xi). \end{aligned}\right\} \text{Taylor series.}$$

Refining:

$$y_1' = y_0' + \frac{h}{2}(y_0'' + y_1''), \text{ trapezoidal formula, error} = -\frac{h^3}{12}y^{(4)}(\xi),$$

$$y_1 = y_0 + \frac{h}{2}(y_0' + y_1') + \frac{h^2}{12}(y_0'' - y_1''), \text{ error} = \frac{h^5}{720}y^{(5)}(\xi).^9$$
$$x_0 \le \xi \le x_1.$$

2-Point Schedule

Predicting:

$$y_2' = \frac{120}{h}(y_1 - y_0) - 55y_0' - 64y_1' + 2h(-4y_0'' + 7y_1''), \text{ error} = \frac{h^5}{20}y^{(6)}(\xi),$$

$$y_2 = -31y_0 + 32y_1 + 2h(-7y_0' - 8y_1') + 2h^2(-y_0'' + 2y_1''), \text{ error} = \frac{h^6}{90}y^{(6)}(\xi).$$

Refining:

$$y_2' = \frac{1}{11}\left\{\frac{240}{h}(y_1 - y_0) - 101y_0' - 128y_1' + h(-13y_0'' + 40y_1'' + 3y_2'')\right\},$$

$$\text{error} = -\frac{8}{3465}h^6y^{(7)}(\xi_1) - \frac{1}{9240}h^7y^{(8)}(\xi_2) - \frac{1}{7\,48440}h^8y^{(9)}(\xi_3),$$

$$y_2 = 2y_1 - y_0 + \frac{3h}{8}(y_2' - y_0') + \frac{h^2}{3}\left[-\frac{1}{8}(y_0'' + y_2'') + y_1''\right], \text{ error} = \frac{h^8}{60480}y^{(8)}(\xi),^{10}$$
$$x_0 \le \xi \le x_2,$$
$$x_0 \le \xi_i \le x_2,\ i = 1,2,3.$$

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[9] See (8, p. 25).

[9] See (8, p. 76). This last 1½-point hyperosculatory formula is identical with the 2-point osculatory quadrature formula given at the end of the 1-point schedule in the third section.

[10] This formula occurs also in (6, p. 232).

3-Point Schedule

Predicting:

$$y_2' = \frac{1}{h}\left(\frac{2511}{4}y_{-1} - 1728y_0 + \frac{4401}{4}y_1\right) + \frac{895}{4}y_{-1}' - 243y_0' - \frac{1809}{4}y_1'$$
$$+ h\left(\frac{93}{4}y_{-1}'' - 270y_0'' + \frac{243}{4}y_1''\right), \text{ error} = \frac{11}{3360}h^8y^{(9)}(\xi),$$

$$y_2 = \frac{245}{2}y_{-1} - 351y_0 + \frac{459}{2}y_1 + h\left(\frac{87}{2}y_{-1}' - 54y_0' - \frac{189}{2}y_1'\right)$$
$$+ h^2\left(\frac{9}{2}y_{-1}'' - 54y_0'' + \frac{27}{2}y_1''\right), \text{ error} = \frac{h^9}{1680}y^{(9)}(\xi).$$

Refining:

$$y_2' = \frac{1}{157}\left\{\frac{81}{h}[103y_{-1} - 416y_0 + 313y_1] + 2830y_{-1}' - 8451y_0' - 11232y_1' + 3h[92y_{-1}''\right.$$
$$\left. - 1557y_0'' + 630y_1'' + 11y_2'']\right\}, \text{ error} = -\frac{103}{13\,18800}h^9y^{(10)}(\xi_1)$$
$$- \frac{1}{4\,39600}h^{10}y^{(11)}(\xi_2) - \frac{1}{580\,27200}h^{11}y^{(12)}(\xi_3),$$

$$y_2 = y_{-1} + \frac{1}{103}[729(y_1 - y_0) + 3h[11(y_2' + y_{-1}') - 81(y_1' + y_0')]$$
$$+ 3h^2[y_{-1}'' - y_2'' + 27(y_1'' - y_0'')]], \text{ error} = \frac{h^{11}}{31\,72400}y^{(11)}(\xi),$$

$$x_{-1} \leq \xi \leq x_2,$$
$$x_{-1} \leq \xi_i \leq x_2,\ i = 1,2,3.$$

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Electronic Monitoring Instrument.—Measuring and recording nuclear radiation and answering 12 important questions in 15 seconds on radioactive contamination is the job being done by a new electronic device developed by General Electric Company engineers at the Hanford atomic plant here.

The instrument electronically scans the hands and shoes of workers, seeking out tiny bits of radioactive contamination which may have been accidentally brought out of adjacent radiation work areas. It detects alpha, beta and gamma radiation as against previous devices which spotted either alpha radiation alone or beta and gamma radiation combined.

The 12 questions it answers regard types of radiation from specific areas of hands and shoes. Six circuits are geared to pick up evidence of beta-gamma radiation on six areas of hands and shoes, while the remaining six detect tell-tale alpha radiation from the same area.

If a minute bit of radioactive material is discovered, the machine flashes a red warning light marked, for example, "alpha left palm" or "beta-gamma, left sole." Once discovered, the contamination can be safely removed.

Green lights flashing on at the end of the counting period indicate there is no radioactivity above the preset or "background" sensitivity levels of the instrument.

Members of General Electric's Hanford Laboratories Operation increased the "brain power" of the new detection instrument by using scintillation probes for detecting alpha radiation, and Geiger-Mueller tubes for revealing the presence of beta and gamma rays. Both detection systems are combined in the same probes.

The scintillation portions of the probes use zinc sulfide powder as the scintillator phosphor and when alpha particles strike the powder they create tiny flashes of light. Multiplier phototubes pick up the faint light flashes and send electrical current in proportion to the strength of the flashes to the signal lights.

The beta-gamma warning lights on the outside panel light up when activated by current created when beta or gamma rays cause ionization in glasswall Geiger-Mueller tubes.

If no radiation is detected which would activate the warning lights, a network of relays turns on the 12 green "safe" lights.

The new instrument's intelligence quotient is nearly matched by its economical feasibility. When placed in production, the new instrument will combine into one operation the two steps required of old "four-fold" instruments, the hand and shoe count mainstay of the past decade.

The new machine performs the survey in half the time of the old instruments and will cost less to manufacture than two of the old machines.

All channels or circuits have been designed for individual installation and can be easily removed for repair, a feature expected to reduce maintenance costs and trim the "down-time" charged against the four-fold and five-fold instruments.

The scintillation probes in the new Hanford instrument also are expected to resist humidity and temperature change more effectively than the air-chamber probes in the old counters.

LATEST DEVELOPMENTS IN OFF-THE-ROAD LOCOMOTION

BY

M. G. BEKKER[1]

The field of "Off-the-Road Locomotion" is one of the oldest known to man and yet for may years its problems attracted little attention among the engineering sciences. Prior to and after World War II only a few companies, manufacturing excavation and earthmoving equipment, have made sporadic efforts in this area.

During World War II, with mechanized tactics spreading from static trench warfare over fields, mountains and through swampy country, all belligerents were abruptly confronted with insurmountable problems of bogged-down transportation.

Stemming from research initiated at that time, a new applied mechanics has been developing. Research so far performed has discovered a number of basic concepts and has indicated new and more promising avenues of approach.

In recognition of the whole problem, a Land Locomotion Research Laboratory has been established at the Ordnance Tank-Automotive Command and the University of Michigan has introduced a new curriculum for automotive engineers including courses in Land Locomotion Mechanics. More cooperation with industry and the universities in this new field is already in sight. The highlights of the present "state of the art" may be briefly described as follows.

The problem of the extent to which locomotion is possible in a given terrain is primarily defined by the ability of soil to provide sufficient resistance to bear the gravity loads and to produce the necessary thrust.

In order to determine the respective values, it is necessary to develop methods which would relate weight and thrust of a vehicle to certain mechanical properties of soil in a manner generally similar to that in which fluid mechanics, for instance, relates lift and drag of a ship to the properties of the air or water. In addition, the effect of terrain roughness upon the speed of motor vehicles must be considered.

The whole problem may be approached from two viewpoints: (*a*) from a long-range viewpoint in which more rigorous scientific solutions could be made available, and (*b*) from a short-range point of view, in which less accurate, although more immediate, engineering solutions could be arrived at.

The first approach has been based on a study of deformation and strength of the ground by an application, whenever possible, of the

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modern theories of plasticity. The work done in this field with reference to land locomotion (1, 2)[2] indicates that such an approach may become the basis for a physical definition of weight/thrust relationships in cross-country locomotion if a number of difficulties are overcome (3).

The study of the effect of terrain roughness, when conducted in a manner similar to that in which naval architects study random wave effects of rough seas upon ship motion (4) defining vibrations of a vehicle within a "response spectrum," may also lead to a general solution of vehicle morphology. If one assumes that vehicle vibrations are one of the most serious factors in slowing down cross-country locomotion to a few miles per hour (5), then the treatment of the problem, in the same way in which mobility on the sea has been approached, becomes an essential part of research in Land Locomotion.

All this points to rather distant goals and solutions. However, a number of promising approaches to the mechanics of land locomotion made in recent years from a short range viewpoint seem to offer a fairly good chance of a more immediate success, at least from an engineering angle, which may be recognized as useful until such time when more rigorous solutions are made available.

In 1944, E. W. E. Micklethwait (6) called attention to the fact that the tractive forces of a vehicle and its ability to stay on the surface of the ground can be estimated by Coulomb's equation which expresses shear as the function of cohesion, pressure and friction, and by the bearing capacity formula such as that proposed by Prandtl, and Terzaghi (7).

In an independent work the writer (8) followed the same direction. As a result of these attempts, the present concept of crossing a soft soil within the limits of the bearing capacity of the ground may be expressed by the following approximate equations (9):

$$T_g = lbc + W_s \tan \phi \tag{1}$$

$$W_s = lb(N_c c + \gamma b N_\gamma) \tag{2}$$

where T_g is soil thrust, or so called gross tractive effort; W_s is the safe load which is determined by the bearing capacity of the ground; l is the length and b the width of the ground contact area; c and ϕ are the values of cohesion and friction respectively, or may be considered the coefficients of Coulomb's equation; N_c and N_γ are Terzaghi's bearing capacity factors (see ref. 7); γ is the specific gravity of soil.

Equation 1 does not include the effect T_g' of spuds or tread, which may augment the thrust by the following approximate value (10):

$$T_g' = 2hlc + W \tan \phi \left\{1 + 0.64 \left[\frac{h}{b} \cot^{-1}\left(\frac{h}{b}\right)\right]\right\} \tag{3}$$

[2] The boldface numbers in parentheses refer to the references appended to this paper.

where h is the spud or lug height. As this effect is rather limited for conventional vehicles (see ref. 9) Eq. 3 will not be considered in the present reasoning which is concerned with general evaluation of trends and orders of magnitude rather than detailed solutions.

If Eq. 1 is divided by Eq. 2 then the weight/thrust ratio of a vehicle safely "floating" on the ground surface will be expressed as follows:

$$\frac{T_g}{W_s} = \frac{c}{N_c + \gamma b N_\gamma} + \tan \phi. \tag{4}$$

This illustrates that, for an average arable soil, the thrust/weight ratio of the "high flotation" vehicle operating on the ground surface is

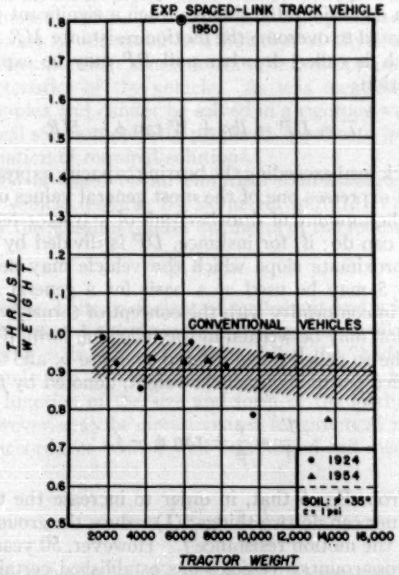

FIG. 1. 1924–1954 progress in soil-tractor relationship.

constant and is practically independent of vehicle design within the assumed premises.

This is confirmed by experimental evidence: Fig. 1 shows the drawbar pull/weight ratio of agricultural tractors produced for the past 30 years. That ratio displays an average constant value, enclosed between

0.85 – 0.95 in a soil which was assumed in the present case ($\phi = 35°$; $c = 1$ psi.).

Figure 1 also shows that the scatter of points is today the same as it was in 1924, which indicates that the considered tractors not only failed to improve their pull for so many years, but also that the difference between the most and least successful tractor today is the same as it was a few decades ago. Equations 1 and 2 explain why there has been little progress in pulling more load with less weight: this situation is a consequence of the present physical relationship between the soil and the vehicle that results from the application of the concept of "high flotation."

If one contemplates a "low flotation" vehicle which has surpassed the bearing capacity of the ground as expressed by Eq. 2, that is, which is moving with a considerable sinkage, then a significant portion of soil thrust T_s is wasted to overome the motion resistance MR and the useful thrust T which is called drawbar pull DP may be expressed by the following equation:

$$T = DP = lbc + W \tan \phi - MR \tag{5}$$

where W is track load exceeding the bearing capacity expressed by Eq. 2.

Equation 5 expresses one of the most general values of vehicle performance, as the amount of drawbar pull of a tractor tells how much useful work it can do; if, for instance, DP is divided by weight it expresses an approximate slope which the vehicle may negotiate. Accordingly, Eq. 5 may be used as a basis for a general evaluation of present trends in conformity with the concept of thrust/weight ratio.

This equation may be written in the following form, if it is assumed that W/bl is the so called mean ground pressure p, and MR/W is the motion resistance per unit of vehicle weight, denoted by f:

$$\frac{T}{W} = \frac{c}{p} + \tan \phi - f. \tag{6}$$

It is seen from Eq. 6 that, in order to increase the thrust/weight ratio, the designer can do two things: (1) reduce the ground pressure p, and (2) reduce the motion resistance f. However, 50 years' experience in design of cross-country vehicles has established certain limitations which so far could not be avoided. These rigid landmarks of the existing design concepts say that, for heavy-weight tracked vehicles, it is extremely difficult to get less "nominal" ground pressure, p, than 15 psi.; the middleweight class is between 5–10 psi., whereas the lightweight vehicles usually cannot be designed with less ground pressure than approximately 3 psi.

These limits have been already achieved and as no more reduction

in ground pressure is possible, they constitute the end of what the designer may do. It also results from Eq. 6 that in frictional soils, where $c = 0$, the ground pressure below the bearing capacity value does not affect the gross thrust/weight ratio since the latter is produced solely by the constant value (for a given soil) of tan ϕ.

Thus present design concepts of crawler vehicles lead to performances rather rigidly limited as long as the gross thrust is determined directly by Coulomb's expression $(c/p + \tan \phi)$, and the only other way of radically increasing the net thrust/weight ratio within the existing design concepts is to decrease the motion resistance f. Since Eq. 6 is quite general, this conclusion refers to both tracked and wheeled behicles.

Let it be seen, accordingly, what can be done in reducing motion resistance of motor vehicles in a cross-country operation. First, however, f-value must be expressed as a function of physical properties of soils which determine their stress-strain relationship, and as a function of the characteristics of the vehicle. As was mentioned before, the problem is complex and cannot be solved in a rigorous way. However, a semi-empirical solution recently proposed appears to be useful for the first approximation of required solutions.

The method is based on an empirical load-sinkage function, first considered in vehicle research by Bernstein in 1913 (11) and later on generalized by the Russian tractor engineers in the following form (12):

$$p = kz^n \tag{7}$$

where z is sinkage, k the coefficient of settlement and n is the exponent of sinkage. For $n = 1$ this formula is reduced to a well known form in soil mechanics: $p = kz$, where k is often called the "bedding number."

Unfortunately, Eq. 7 is most inconvenient because k is not a "constant" but a function of the size and form of the loading area. This difficulty, however, may be circumvented for practical purposes if it is assumed, in accordance with a well established soil mechanics notion, that (13):

$$p = \left(\frac{k_c}{b} + k_\phi\right) z \tag{8}$$

where k_c is the "cohesive" coefficient of soil deformation, while k_ϕ is the "frictional" one; b as previously shown means the width (smaller dimension) of the loading area, or the diameter of that area.

Hence, by combining Eqs. 7 and 8 the stress-strain relationship of granular soil mass may be expressed in the following form (15):

$$p = \left(\frac{k_c}{b} + k_\phi\right) z^n. \tag{9}$$

Since all the values k and n may be readily determined by an experiment (15), this equation so far appears to be very convenient in vehicle mobility research.

Studies performed in this respect indicate that the total resistance to vehicle motion may be split into three components (see ref. 10): (*a*) soil compaction; (*b*) bulldozing resistance; and (*c*) lateral drag.

The first type of resistance is defined by an integral of the elementary work of forces which compact the ground to the given depth: $\int pdz$.

Assuming uniform load distribution along a rectangular contact area of width b, this work performed at a distance equal to the length l of that area may be defined by the following equation:

$$f_c = \frac{W^{1/n}}{(n+1)(k_c + bk_\phi)^{1/n} l^{(n+1)/n}}. \tag{10}$$

The form of this equation is general and also appears applicable to low pressure pneumatic tires. An equation similar in structure was proposed for rigid wheels by Bernstein in 1913 (11).

Bulldozing resistance f_b may be expressed in terms of mechanical properties of the ground if it is assumed equal to the passive earth pressure of a wall, as defined in soil mechanics. When adapting to this purpose the equation proposed by Terzaghi (see ref. 7), a simplified formula which expresses f_b may be written as follows:

$$f_b = \frac{b \sin(\alpha + \phi)}{2\ W \sin\alpha \cos\phi} (2czK_c + \gamma z^2 K_\gamma) \tag{11}$$

where K_c and K_γ are pure numbers which are functions of previously quoted values N_c and N_γ and of the angle of friction ϕ (see ref. 7). Angle α is the "angle of approach" of the track or tire equal to the slope of the corresponding retaining wall which, it is assumed, sustains the passive earth pressure of bulldozing.

The lateral drag resistance is of a more complex nature, requires a more empirical treatment (see ref. 9), and may be expressed as a linear function of f_b. It may be neglected in the present consideration for the sake of simplicity; the conclusions reached in respect to vehicle performance will be on the optimistic side as we forego one component of resistance to motion. However, the general trend of the evaluation will remain unchanged. Consider what can be deduced from Eqs. 6, 10 and 11 which approximately express the net thrust/weight ratio in the following form, if it is assumed that $W = pbl$, and $p = (k_c/b + k_\phi)\, z^n$:

$$\frac{T}{W} = \left(\frac{c}{p} + \tan\phi\right) - \frac{b^{1/n}}{l}\left\{\frac{l^{1/n}}{(n+1)l^n}\left(\frac{p}{k_c + bk_\phi}\right)^{1/n} + \left(\frac{p}{k_c + bk_\phi}\right)^{1/n} \times \frac{\sin(\alpha+\phi)}{2p \sin\alpha \cos\phi}\left[2cK_c + \gamma K_\gamma \left(\frac{pb}{k_c + bk_\phi}\right)^{1/n}\right]\right\}. \tag{12}$$

As was mentioned before, the Coulombian expression enclosed in the first pair of brackets cannot be expected to increase radically the thrust/weight ratio, and the only hope for improvement is to reduce the resistance value of the expression enclosed in the second set of brackets.

This part of Eq. 12 indicates that width b of the track and track length l have considerable effect upon motion resistance. In general, the smaller b and the larger l, the lower the motion resistance. But any improvement in this respect again cannot go beyond the presently achieved standards because narrow, long tracks would lead to an unsteerable vehicle. Hence, no radical progress can be expected in this area, and since p cannot be reduced beyond existing values, the checkmate appears complete.

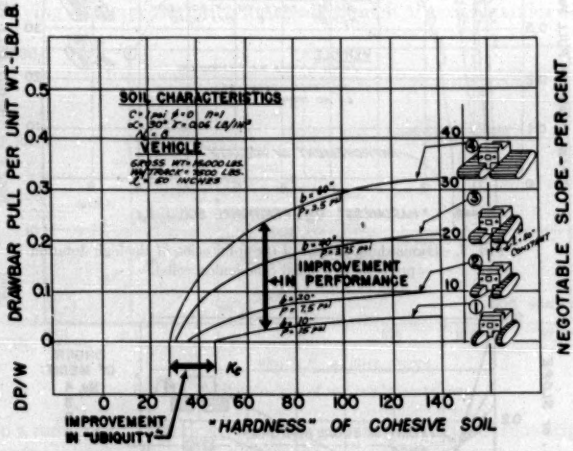

FIG. 2. Diminishing returns of the application of the high flotation principle (cohesive frictionless soils).

That the present trend to reduce ground pressure through the design of wider and wider tracks cannot lead very far, may be shown in another way. If Eq. 12 is plotted in terms of $k = k_c/b + k_\phi$ in two versions, one assuming $c = 0$, $k_c = 0$ and the other assuming $\phi = 0$, $k_\phi = 0$, then two graphs as shown in Figs. 2 and 3 may be obtained (where $T/W = DP/W$). They clearly indicate that an increase of b beyond certain limits does not pay, since it leads to the rapid decrease of negotiable k-values, which determine what may be called "ubiquity," as these values describe how far a "weak" ground may be negotiated before it immobilizes the vehicle.

If it is assumed that a vehicle operates in spring thaw, or autumn rains which produce a soft top layer gradually thickening into a hard bottom, then Eq. 12, applied to such specific conditions, indicates the critical mud depth z which may be negotiated by the given vehicle.

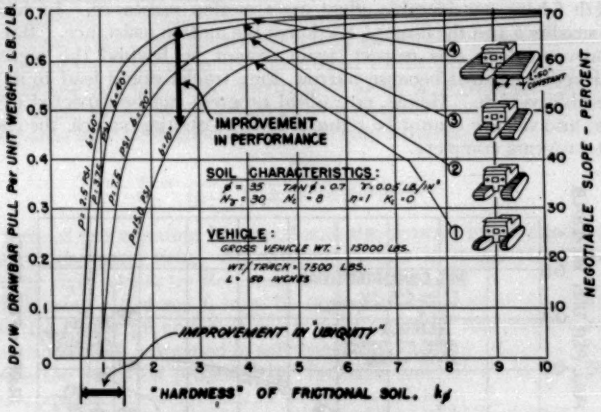
DP/W DRAWBAR PULL Per UNIT WEIGHT - LB./LB.
NEGOTIABLE SLOPE - PERCENT
IMPROVEMENT IN PERFORMANCE
SOIL CHARACTERISTICS:
VEHICLE:
GROSS VEHICLE WT. - 15000 LBS.
WT./TRACK = 7500 LBS.
L = 50 INCHES
IMPROVEMENT IN "UBIQUITY"
"HARDNESS" OF FRICTIONAL SOIL, k_ϕ

FIG. 3. Diminishing returns of the application of the high flotation principle (frictional cohesionless soils).

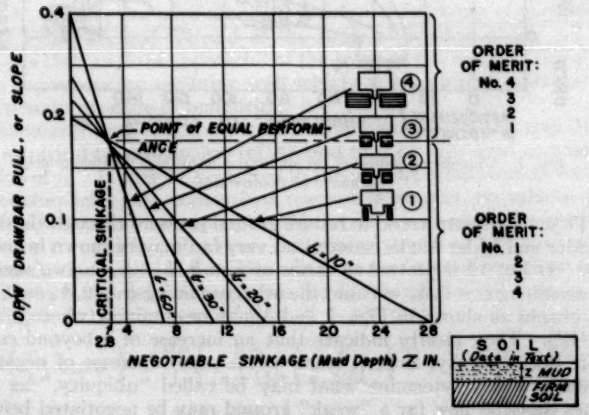

FIG. 4. Mud-crossing performance.

The results computed for the same vehicles as shown in Figs. 2 and 3 are shown in Fig. 4, assuming that only bulldozing resistance caused by the top layer is significant.

It is evident that, in such a type of soil, the wide "high flotation" tracks are absolutely detrimental. The conclusions may not be basically new, as many people concerned with off-road locomotion concede that, despite the enormous technological development, the T/W ratio in mud is entirely insufficient.

It is thus perhaps interesting to note, on the basis of foregoing remarks, that the land locomotion of motor vehicles is practically on the same "level" as the locomotion of animals (Fig. 5). This observation may be considered an indication that a more rational approach to the problem may bring improvement somewhat similar to that in which the art of flying was improved with the establishment of aerodynamics which

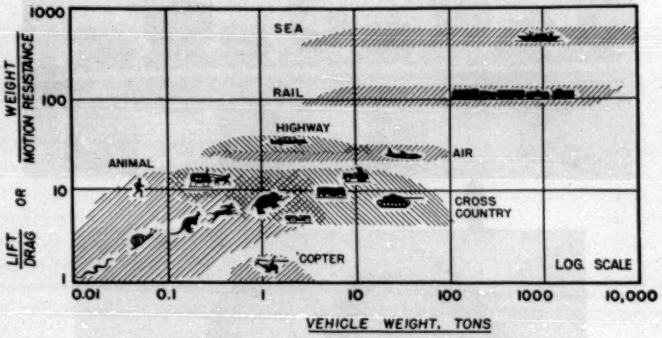

FIG. 5. Relative merits of various locomotions.

led to a reduction of the drag, by a more radical departure from original concepts.

The way to progress seems to be two-fold: one way is to produce a vehicle which would utilize the soil strength necessary for propulsion, not in accordance with phenomena defined directly by Coulomb's criterion (Eq. 12), but in accordance with some other "better" process of soil shear in which Coulombian effects take more indirect place. The second way is to design new tracks and wheels which would encounter less motion resistance than that defined in terms of bulldozing and compaction resistance as expressed by Eqs. 10 and 11, for conventional solutions.

A great deal of research has been performed in this respect (see refs. 5, 8). Although, it may be premature to formulate precise conclusions, the following has been firmly established.

A track composed of cleats spaced apart (Fig. 6) at a strictly determined distance (see ref. 8) produces shear which gives, within certain load limits, a much higher thrust T than that produced by a conventional track (Fig. 7) directly shearing the soil along Coulomb's criterion.

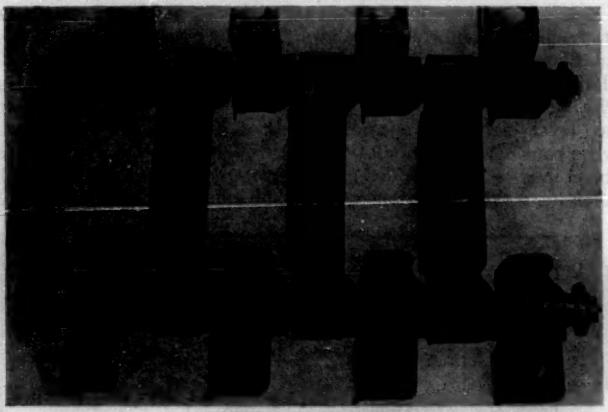

FIG. 6.

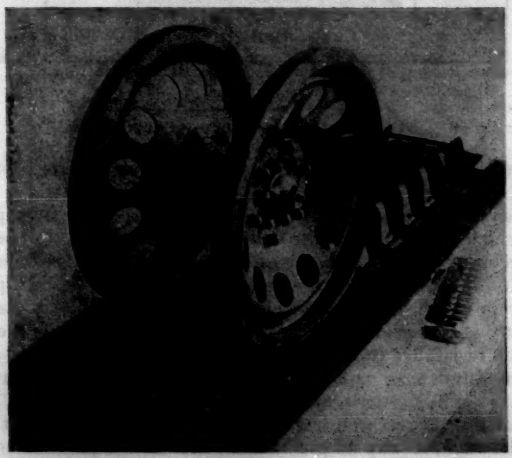

FIG. 7.

Moreover, the bulldozing resistance and the lateral drag resistance of such a "Spaced-Link-Track" (Fig. 6) is smaller than that of a comparable conventional track (Fig. 7).

At this moment, five experimental vehicles of this type have been built and all of them confirm the theoretical expectations. One of these vehicles, the "Groundhog," has doubled, in a variety of soil and snow conditions, the T/W ratio as compared to a conventional vehicle of the same class (14, 8). A commercial agricultural tractor, redesigned along the new principles in Canada, also doubled the draw-bar pull in a sandy soil and snow while preserving the original weight and the over-all dimensions (Fig. 8).

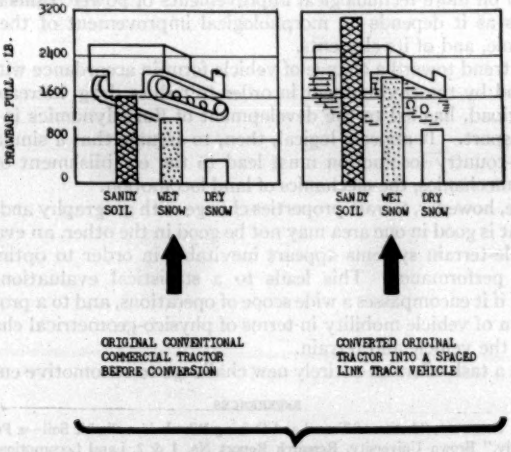

FIG. 8.

Thus, it has been demonstrated that Eq. 12 in its present form may not be the only criterion of vehicle performance in the future although it has been that criterion for the past 50 years; it may be modified, which forecasts a possibility of a more radical improvement. This has been shown in a number of conditions in which conventional vehicles have been pulling only 75–95 per cent of their weight while the new vehicles produce values reaching more than 150 per cent limit in the soil described in Fig. 1.

This was achieved at the cost of a basic change in the prevailing concept of vehicle design. The new ideas seem to contradict the present trends such as the "high flotation" trend which leads to diminishing

returns since the design of spaced-link tracks appears to pay off for most of the design conditions. Since track width b in this case has different meaning than that of a conventional track, the spaced-link track vehicles are being recommended as "bellyless" ones with tracks covering possibly the entire vehicle width, which may not be practical in a conventional solution; in some cases, to take the full advantage of "bellyless" approach, new forms of power trains appear necessary in order to fit them in such a structure (5).

The above few considerations, illustrating the present state of the art and possible future departure from decades of routine, attempt to depict the real nature of the problem: progress does not seem to depend as much on more technological improvements of power trains and suspensions as it depends on morphological improvement of the vehicle as a whole, and of its elements.

The trend toward a change of vehicle form in accordance with forces generated by the environment in order to reduce drag, increase thrust and payload, has led to the development of fluid-dynamics in air and sea transport. It appears logical, then, to assume that a similar trend in cross-country locomotion must lead to the establishment of a new applied mechanics, the mechanics of land locomotion.

Since, however, terrain properties change with geography and climate and what is good in one area may not be good in the other, an evaluation of vehicle-terrain systems appears inevitable in order to optimize the over-all performance. This leads to a statistical evaluation of the problem if it encompasses a wide scope of operations, and to a probability definition of vehicle mobility in terms of physico-geometrical characteristics of the vehicle and terrain.

Such a task offers an entirely new challenge to automotive engineers.

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Electronic Computation of Pipe Stress Problems.—Hundreds of engineering manhours are now being saved by a new high speed method for the automatic electronic computation of chemical and process industry pipe stress problems. The valuable engineering talent that hitherto has been spent in tedious calculation of the many displacements and stresses in the complex piping systems of chemical processing and power plants can now be devoted to more creative effort (thus helping to alleviate the technical manpower shortage) while electronic computers perform the "dog work." This permits the design and analysis of alternative piping systems so that configurations can be found.

Dr. Leslie G. Peck and his colleagues at Arthur D. Little, Inc., industrial research company of Cambridge, Mass., developed this breakthrough in engineering calculations, called the "6 × 6 flexibility method," jointly with the Blaw Knox Company of Pittsburgh. This mathematical technique makes possible the machine solution of problems of unlimited complexity. More than fifty complex systems in the power piping field, as well as several intricate chemical piping systems, have already been handled with this new technique.

Over the past two years, a team of mathematicians, mechanical engineers, and piping specialists from Arthur D. Little, Inc., and Blaw Knox Company developed the mathematical systems necessary for the electronic computation of forces, moments, displacements, rotations, and stresses of the "members" of piping systems. Every member (sections of straight or curved pipes, anchors, restraints, valves, and other rigid pieces of equipment) has its own response to temperature changes, and these responses must be calculated accurately to be sure of adequate safety factors. Figuring all these data "by hand" takes up immense amounts of expensive engineering time, while a large electronic computer can perform the same operation very rapidly after the problem has been properly programmed. The program developed is so general it can use information on an isometric drawing of a piping system with practically no preparatory hand calculation.

The 6 × 6 flexibility method is the fastest, most economical, and most accurate method in existence for this work. The result of the machine's calculation is a printed form that shows for each point of the system the accurate numerical values of forces, moments, displacements, rotations, and stresses. Moreover, the machine checks the accuracy of the initial information, makes corrections where required, and prints out appropriate information if there are inconsistencies. The cost of these computations is proportional to the number of members in the system. Since the intermediate results of the calculations, such as shape coefficients, are saved on magnetic tape, problems can be rerun with small modifications at a fraction of the original cost. This facilitates designing piping systems by testing alternates.

ON A "BEST" 9-POINT DIFFERENCE EQUATION ANALOGUE OF LAPLACE'S EQUATION

BY

DONALD GREENSPAN [1]

A well known technique[2] for the numerical solution of a Dirichlet problem involves replacement of the Laplace equation

$$u_{xx} + u_{yy} = 0 \tag{1}$$

by the "9-point" difference equation

$$-20u_0 + 4(u_1 + u_2 + u_3 + u_4) + u_5 + u_6 + u_7 + u_8 = 0. \tag{2}$$

The primary objective of this paper is to show, subject to the assumptions and definitions below and using only $u_0, u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8$, that Eq. 2 is a "best" 9-point analogue of Eq. 1. The general method will involve substitution of Taylor series into the expression $L_9(u) = \sum_0^9 \alpha_i u_i$, and by use of the fact that u is harmonic, determination of the α_i such that $\sum_0^9 \alpha_i u_i + o(h^n) \equiv 0$. Discarding the terms $o(h^n)$ will lead to the difference approximation. For Eq. 2, it will be shown that $n = 8$.

It is assumed that the "grid constant" h is positive. The following notation will be used throughout:

$$\begin{aligned} u_0 &= u(x, y), \quad u_1 = u(x + h, y), \quad u_2 = u(x, y + h), \\ u_3 &= u(x - h, y), \quad u_4 = u(x, y - h), \\ u_5 &= u(x + h, y + h), \quad u_6 = u(x - h, y + h), \\ u_7 &= u(x - h, y - h), \quad u_8 = u(x + h, y - h). \end{aligned} \tag{3}$$

$$\frac{\partial^{m+n} u}{\partial x^m \partial y^n} = u_{m,n} \tag{4}$$

$$A_{m,n} = \frac{u_{m,n}}{m!n!}. \tag{5}$$

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[2] See, for example, ref. 5 of the appended bibliography, Chapters VIII and X for a complete discussion of the technique.

The Laplace equation then has the form $u_{2,0} + u_{0,2} = 0$. Moreover, for harmonic functions, $u_{m+2,n} + u_{m,n+2} = 0$; m, n are non-negative integers. The latter relation may be expressed in terms of the A's by:

$$(m+2)(m+1)A_{m+2,n} + (n+2)(n+1)A_{m,n+2} = 0, \tag{6}$$

from which it follows that

$$\begin{array}{lll}
A_{2,0} + A_{0,2} = 0 & A_{2,3} + 10A_{0,5} = 0 & A_{3,4} + 5A_{1,6} = 0 \\
3A_{3,0} + A_{1,2} = 0 & 15A_{6,0} + A_{4,2} = 0 & A_{2,5} + 21A_{0,7} = 0 \\
A_{2,1} + 3A_{0,3} = 0 & 10A_{5,1} + 3A_{3,3} = 0 & 28A_{8,0} + A_{6,2} = 0 \\
6A_{4,0} + A_{2,2} = 0 & A_{4,2} + A_{2,4} = 0 & 7A_{7,1} + A_{5,3} = 0 \\
A_{3,1} + A_{1,3} = 0 & 3A_{3,3} + 10A_{1,5} = 0 & 5A_{6,2} + 2A_{4,4} = 0 \\
A_{2,2} + 6A_{0,4} = 0 & A_{2,4} + 15A_{0,6} = 0 & A_{5,3} + A_{3,5} = 0 \\
10A_{5,0} + A_{3,2} = 0 & 21A_{7,0} + A_{5,2} = 0 & 2A_{4,4} + 5A_{2,6} = 0 \\
2A_{4,1} + A_{2,3} = 0 & 5A_{6,1} + A_{4,3} = 0 & A_{3,5} + 7A_{1,7} = 0 \\
A_{3,2} + 2A_{1,4} = 0 & 5A_{5,2} + 3A_{3,4} = 0 & A_{2,6} + 28A_{0,8} = 0 \\
 & 3A_{4,3} + 5A_{2,5} = 0 &
\end{array} \tag{7}$$

One may give (7) equivalently by having only terms of form $A_{i,j}$, $i = 0, 1$; $j = 2, 3, 4, 5, 6, 7, 8$ on the right hand sides as follows:

$$\begin{array}{lll}
A_{2,0} = -A_{0,2} & A_{2,3} = -10A_{0,5} & A_{3,4} = -5A_{1,6} \\
A_{3,0} = -(1/3)A_{1,2} & A_{6,0} = -A_{0,6} & A_{2,5} = -21A_{0,7} \\
A_{2,1} = -3A_{0,3} & A_{5,1} = A_{1,5} & A_{8,0} = A_{0,8} \\
A_{4,0} = A_{0,4} & A_{4,2} = 15A_{0,6} & A_{7,1} = -A_{1,7} \\
A_{3,1} = -A_{1,3} & A_{3,3} = -(10/3)A_{1,5} & A_{6,2} = -28A_{0,8} \\
A_{2,2} = -6A_{0,4} & A_{2,4} = -15A_{0,6} & A_{5,3} = 7A_{1,7} \\
A_{5,0} = (1/5)A_{1,4} & A_{7,0} = -(1/7)A_{1,6} & A_{4,4} = 70A_{0,8} \\
A_{4,1} = 5A_{2,5} & A_{6,1} = -7A_{0,7} & A_{3,5} = -7A_{1,7} \\
A_{3,2} = -2A_{1,4} & A_{5,2} = 3A_{1,6} & A_{2,6} = -28A_{0,8} \\
 & A_{4,3} = 35A_{0,7} &
\end{array} \tag{8}$$

Now let

$$L_0(u) = \sum_0^9 \alpha_i u_i. \tag{9}$$

By

(*a*) noting $u_0 = u(x, y) = u_{0,0}$,
(*b*) expressing each u_j, $j = 1, 2, 3, 4, 5, 6, 7, 8$, in a Taylor series about (x, y),
(*c*) substituting the results of (*a*) and (*b*) into Eq. 9,
(*d*) using the relationships of Eq. 5,

it results that Eq. 9 can be expressed as follows:

$$
\begin{aligned}
L_9(u) = {} & A_{0,0}(\alpha_0+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8) \\
& +h[A_{1,0}(\alpha_1-\alpha_2+\alpha_3-\alpha_4-\alpha_5+\alpha_6+\alpha_7-\alpha_8)+A_{0,1}(\alpha_1-\alpha_4+\alpha_5+\alpha_6-\alpha_7-\alpha_8)] \\
& +h^2[A_{2,0}(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8)+A_{1,1}(\alpha_1-\alpha_2+\alpha_3-\alpha_4+\alpha_5-\alpha_6+\alpha_7-\alpha_8)+A_{0,2}(\alpha_1+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8)] \\
& +h^3[A_{3,0}(\alpha_1-\alpha_2+\alpha_3-\alpha_4-\alpha_5+\alpha_6+\alpha_7-\alpha_8)+A_{2,1}(\alpha_1+\alpha_2-\alpha_5+\alpha_6-\alpha_7-\alpha_8)+A_{1,2}(\alpha_1-\alpha_2+\alpha_3-\alpha_4-\alpha_5+\alpha_6+\alpha_7+\alpha_8) \\
& \quad +A_{0,3}(\alpha_1-\alpha_4+\alpha_5+\alpha_6-\alpha_7-\alpha_8)] \\
& +h^4[A_{4,0}(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8)+(A_{3,1}+A_{1,3})(\alpha_1-\alpha_2+\alpha_3-\alpha_4-\alpha_5+\alpha_6+\alpha_7-\alpha_8) \\
& \quad +A_{2,2}(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8)+A_{0,4}(\alpha_1+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8)] \qquad (10) \\
& +h^5[A_{5,0}(\alpha_1-\alpha_2+\alpha_3-\alpha_4-\alpha_5+\alpha_6+\alpha_7-\alpha_8)+(A_{4,1}+A_{2,3})(\alpha_1+\alpha_2-\alpha_5+\alpha_6-\alpha_7-\alpha_8) \\
& \quad +(A_{3,2}+A_{1,4})(\alpha_1-\alpha_2-\alpha_5+\alpha_6-\alpha_7+\alpha_8)+A_{0,5}(\alpha_1-\alpha_4+\alpha_5+\alpha_6-\alpha_7-\alpha_8)] \\
& +h^6[A_{6,0}(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8)+(A_{5,1}+A_{3,3}+A_{1,5})(\alpha_1-\alpha_2+\alpha_3-\alpha_4+\alpha_5-\alpha_6+\alpha_7-\alpha_8) \\
& \quad +(A_{4,2}+A_{2,4})(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8)+A_{0,6}(\alpha_1+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8)] \\
& +h^7[A_{7,0}(\alpha_1-\alpha_2+\alpha_3-\alpha_4-\alpha_5+\alpha_6+\alpha_7-\alpha_8)+(A_{6,1}+A_{4,3}+A_{2,5})(\alpha_1+\alpha_2-\alpha_5+\alpha_6-\alpha_7-\alpha_8) \\
& \quad +(A_{5,2}+A_{3,4}+A_{1,6})(\alpha_1-\alpha_2-\alpha_5-\alpha_6+\alpha_7+\alpha_8)+A_{0,7}(\alpha_1-\alpha_4+\alpha_5+\alpha_6-\alpha_7-\alpha_8)]+o(h^8).
\end{aligned}
$$

However, using (8), Eq. 10 may be rewritten:

$$
\begin{aligned}
L_9(u) = {} & A_{0,0}(\alpha_0+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8) \\
& +h[A_{1,0}(\alpha_1-\alpha_2+\alpha_3-\alpha_4-\alpha_5+\alpha_6+\alpha_7-\alpha_8)+A_{0,1}(\alpha_1-\alpha_4+\alpha_5+\alpha_6-\alpha_7-\alpha_8)] \\
& +h^2[A_{1,1}(\alpha_1-\alpha_2+\alpha_3-\alpha_4-\alpha_5+\alpha_6+\alpha_7-\alpha_8)+A_{0,2}(\alpha_1+\alpha_4-\alpha_5-\alpha_6+\alpha_7+\alpha_8)] \\
& +h^3[(1/3)A_{1,2}(-\alpha_1+\alpha_2+2\alpha_3-2\alpha_4-2\alpha_5+2\alpha_6+2\alpha_7+2\alpha_8)+A_{0,3}(\alpha_1-\alpha_4-2\alpha_5+2\alpha_6+2\alpha_7+2\alpha_8)] \qquad (11) \\
& +h^4[A_{0,4}(\alpha_1+\alpha_2+\alpha_3+\alpha_4-4\alpha_5-4\alpha_6-4\alpha_7-4\alpha_8)] \\
& +h^5[A_{0,5}(\alpha_2-\alpha_4-4\alpha_5-4\alpha_6+4\alpha_7+4\alpha_8)+(1/5)A_{1,4}(\alpha_1-\alpha_2-4\alpha_5+4\alpha_6+4\alpha_7-4\alpha_8)] \\
& +h^6[A_{0,6}(-\alpha_1+\alpha_2-\alpha_3+\alpha_4)+(4/3)A_{1,5}(-\alpha_1+\alpha_2-\alpha_3+\alpha_4)] \\
& +h^7[A_{0,7}(\alpha_2-\alpha_4+8\alpha_5+8\alpha_6-8\alpha_7-8\alpha_8)+(1/7)A_{1,6}(-\alpha_1+\alpha_2-8\alpha_5+8\alpha_6+8\alpha_7-8\alpha_8)]+o(h^8).
\end{aligned}
$$

In order to make Eq. 11 of as high an order in h as possible, consider the set of equations which results from setting the coefficients of each $A_{i,j}$, $i = 0, 1$; $j = 0,1,2,3,4,5,6,7$, in Eq. 11 equal to zero.

The solution of this system of equations is

$$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 4\alpha_5 = 4\alpha_6 = 4\alpha_7 = 4\alpha_8 = -\ (1/5)\alpha_0.$$

If $\alpha_0 = -\ 20$, then $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 4$, $\alpha_5 = \alpha_6 = \alpha_7 = \alpha_8 = 1$.

Hence Eq. 9 becomes

$$L_9(u) = -\ 20u_0 + 4(u_1 + u_2 + u_3 + u_4) + u_5 + u_6 + u_7 + u_8 \qquad (12)$$

and Eq. 11 becomes

$$L_9 = o(h^8). \qquad (13)$$

From Eqs. 12 and 13, then

$$-\ 20u_0 + 4(u_1 + u_2 + u_3 + u_4) + u_5 + u_6 + u_7 + u_8 - o(h^8) \equiv 0. \qquad (14)$$

Discarding the term $o(h^8)$ in Eq. 14, one readily deduces the desired difference equation

$$-\ 20u_0 + 4(u_1 + u_2 + u_3 + u_4) + u_5 + u_6 + u_7 + u_8 = 0. \qquad (15)$$

Equation 14 is the key equation. However, it must, in general, be studied rather carefully. Suppose, for example, one had set $\alpha_0 = -20h^2$. Then Eq. 14 would have taken the form:

$$-20h^2u_0 + 4h^2(u_1 + u_2 + u_3 + u_4) + h^2(u_5 + u_6 + u_7 + u_8) + o(h^{10}) \equiv 0. \quad (14')$$

The process analogous to that described in going from Eq. 14 to Eq. 15 would yield, in the case of Eq. 14′,

$$-20h^2u_0 + 4h^2(u_1 + u_2 + u_3 + u_4) + h^2(u_5 + u_6 + u_7 + u_8) = 0. \quad (15')$$

It appears that Eq. 15′ results from Eq. 14′ by discarding all terms of order at least 10 in h. But of course Eq. 15′ can be divided by h^2, so that this latter statement is correct but misleading. Such trouble is easily avoided as follows. The only allowable type of difference equation will be one of form

$$\sum_0^8 \alpha_i u_i = 0,$$

where $\lim_{h\to 0} \alpha_i \neq 0$, for at least one value of $i = 0, 1, 2, 3, 4, 5, 6, 7, 8$.

Suppose then that in general one obtains in place of Eq. 14:

$$\sum_0^8 \alpha_i u_i + o(h^n) \equiv 0 \quad (14'')$$

and in place of Eq. 15:

$$\sum_0^8 \alpha_i u_i = 0, \quad (15'')$$

and suppose that Eq. 15″ is an allowable difference equation. Then Eq. 15″ is called an approximation of order $(n - 1)$.

It follows immediately that Eq. 15 is an allowable seventh order approximation. It will now be shown that Eq. 15 is a best approximation in the sense that there does not exist an allowable eighth order approximation. The following theorem establishes this result.

Theorem

There does not exist a 9-point, eighth order approximation to the Laplace equation of the form $\sum_0^8 \alpha_i u_i = 0$, in which $\lim_{h\to 0} \alpha_i \neq 0$ for at least one value of $i = 0, 1, 2, 3, 4, 5, 6, 7, 8$.

Proof. Suppose such an allowable eighth order approximation exists. In the exact fashion that Eq. 11 was established, it follows that

$$
\begin{aligned}
L_h(u) = {} & A_{0,0}(\alpha_0+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8) \\
& +h[A_{1,0}(\alpha_1-\alpha_3+\alpha_5-\alpha_6-\alpha_7+\alpha_8)+A_{0,1}(\alpha_2-\alpha_4+\alpha_5+\alpha_6-\alpha_7-\alpha_8)] \\
& +h^2[A_{1,1}(\alpha_5-\alpha_6+\alpha_7-\alpha_8)+A_{0,2}(\alpha_2+\alpha_4-\alpha_1-\alpha_3)] \\
& +h^3[(1/3)A_{1,2}(-\alpha_1+\alpha_3+2\alpha_5-2\alpha_6-2\alpha_7+2\alpha_8)+A_{2,1}(\alpha_1-\alpha_3-2\alpha_5+2\alpha_6-2\alpha_7+2\alpha_8)] \\
& +h^4[A_{2,2}(\alpha_1+\alpha_3+\alpha_5+\alpha_6-4\alpha_5-4\alpha_6-4\alpha_7-4\alpha_8)] \qquad (16) \\
& +h^5[A_{2,3}(\alpha_3-\alpha_1-4\alpha_5-4\alpha_6+4\alpha_7+4\alpha_8)+(1/5)A_{3,2}(\alpha_1-\alpha_3-4\alpha_5+4\alpha_6+4\alpha_7-4\alpha_8)] \\
& +h^6[A_{3,3}(-\alpha_1+\alpha_3-\alpha_5+\alpha_6)+(4/3)A_{1,3}(-\alpha_5+\alpha_6-\alpha_7+\alpha_8)] \\
& +h^7[A_{3,7}(\alpha_3-\alpha_1+8\alpha_5+8\alpha_6-8\alpha_7-8\alpha_8)+(1/7)A_{7,3}(-\alpha_1+\alpha_3-8\alpha_5+8\alpha_6+8\alpha_7-8\alpha_8)] \\
& +h^8[A_{3,8}(\alpha_1+\alpha_3+\alpha_5+\alpha_6+16\alpha_5+16\alpha_6+16\alpha_7+16\alpha_8)]+o(h^9).
\end{aligned}
$$

Now it follows from (8) that $A_{i,j}$, $i = 0, 1; j = 2, 3, 4, 5, 6, 7, 8$, may be viewed as independent parameters for that set of equations, while the $A_{i,j}$ on the left sides of those equations may be considered dependent variables. Since, moreover, $A_{0,0}$, $A_{0,1}$, $A_{1,0}$ are independent of h, it follows that $A_{i,j}$, $i = 0, 1; j = 0, 1, 2, 3, 4, 5, 6, 7, 8$ are all independent parameters. Hence, if in Eq. 16, the right hand side is to be of order at least 9 in h, then it must follow that:

$$\alpha_0+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8 = \epsilon_1, \qquad \epsilon_1 = o(h^9) \quad (17.1)$$

$$\alpha_1-\alpha_3+\alpha_5-\alpha_6-\alpha_7+\alpha_8 = \epsilon_2, \qquad \epsilon_2 = o(h^8) \quad (17.2)$$

$$\alpha_2-\alpha_4+\alpha_5+\alpha_6-\alpha_7-\alpha_8 = \epsilon_3, \qquad \epsilon_3 = o(h^8) \quad (17.3)$$

$$\alpha_5-\alpha_6+\alpha_7-\alpha_8 = \epsilon_4, \qquad \epsilon_4 = o(h^7) \quad (17.4)$$

$$\alpha_2+\alpha_4-\alpha_1-\alpha_3 = \epsilon_5, \qquad \epsilon_5 = o(h^7) \quad (17.5)$$

$$\alpha_1-\alpha_3-2\alpha_5+2\alpha_6+2\alpha_7-2\alpha_8 = \epsilon_6, \qquad \epsilon_6 = o(h^6) \quad (17.6)$$

$$\alpha_2-\alpha_4-2\alpha_5-2\alpha_6+2\alpha_7+2\alpha_8 = \epsilon_7, \qquad \epsilon_7 = o(h^6) \quad (17.7)$$

$$\alpha_1+\alpha_2+\alpha_3+\alpha_4-4\alpha_5-4\alpha_6-4\alpha_7-4\alpha_8 = \epsilon_8, \qquad \epsilon_8 = o(h^5) \quad (17.8)$$

$$\alpha_2-\alpha_4-4\alpha_5-4\alpha_6+4\alpha_7+4\alpha_8 = \epsilon_9, \qquad \epsilon_9 = o(h^4) \quad (17.9)$$

$$\alpha_1-\alpha_3-4\alpha_5+4\alpha_6+4\alpha_7-4\alpha_8 = \epsilon_{10}, \qquad \epsilon_{10} = o(h^4) \quad (17.10)$$

$$\alpha_2-\alpha_4+8\alpha_5+8\alpha_6-8\alpha_7-8\alpha_8 = \epsilon_{11}, \qquad \epsilon_{11} = o(h^2) \quad (17.11)$$

$$\alpha_1-\alpha_3+8\alpha_5-8\alpha_6-8\alpha_7+8\alpha_8 = \epsilon_{12}, \qquad \epsilon_{12} = o(h^2) \quad (17.12)$$

$$\alpha_1+\alpha_2+\alpha_3+\alpha_4+16\alpha_5+16\alpha_6+16\alpha_7+16\alpha_8 = \epsilon_{13}, \qquad \epsilon_{13} = o(h) \quad (17.13)$$

Consider the sub-system of equations 17.1, 17.2, 17.4, 17.5, 17.7, 17.8, 17.10, 17.11, 17.13. The determinant of this system is 12800, and is therefore not zero. The unique solution of the sub-system is of the form

$$\alpha_i = \pm \frac{\begin{vmatrix} \epsilon_1 \cdots \\ \cdot \quad \cdot \\ \cdot \quad \cdot \\ \epsilon_{13} \cdots \end{vmatrix}}{12800}, \qquad i = 0, 1, 2, 3, 4, 5, 6, 7, 8. \qquad (18)$$

If all ϵ_j, $j = 1, 2, 4, 5, 7, 8, 10, 11, 13$, are zero, the α_i are all zero. If all ϵ_j are not zero, then the α_i have the form:

$$\alpha_i = \pm h \frac{\begin{vmatrix} \rho_1 \cdots \\ \cdot \quad \cdot \\ \cdot \quad \cdot \\ \cdot \quad \cdot \\ \rho_{13} \cdots \end{vmatrix}}{12800}$$

where the ρ_j are either zero, non-zero constants and/or terms of order at least one in h. Hence, it follows that $\lim_{h \to 0} \alpha_i = 0$ for all i.

Since the solution of the sub-system is unique and since it has been assumed that the given system of 13 equations has a solution, it follows immediately that the sub-system's solution is the unique solution of the system. Hence, since $\lim_{h \to 0} \alpha_i = 0$, a contradiction has been reached and the theorem follows.

Finally it may be noted that

(*a*) the analogous definitions and techniques may be used to show that the well known 5-point difference equation approximation:

$$-4u_0 + u_1 + u_2 + u_3 + u_4 = 0$$

is a best third order approximation,

(*b*) consideration of the Poisson equation by the same methods described above presents subtler complexities.

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SLOW SHEARING MOTION OVER A HOLLOW

BY

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SUMMARY

The slow steady two-dimensional motion of a viscous liquid over a hollow in an infinite plane is here considered, the flow being that of uniform shear except for the disturbance caused by the hollow. A first approximation to the solution is found by neglecting the inertia terms in the equations of motion. Partial allowance for the effect of inertia can be made, thus obtaining a second approximation to the stream function. An important elastic analogy is also pointed out.

INTRODUCTION

Although the problem of viscous shear flow over a hollow is of great interest and importance in a number of scientific fields, not much consideration has been given to its solution. In theory, we simply set up the equations of motion of the fluid, and attempt to satisfy them together with the kinematical condition that the velocity vanishes on the fixed boundary. The integration of these equations of motion subject to the stated boundary condition is usually impossible for any specified boundary shape; however by making some approximations a solution can be obtained for certain profiles. We here consider a steady two-dimensional incompressible flow and, in the first instance, we assume that the Reynolds number is so small that the inertia terms may be regarded as negligible. The stream function thus satisfies the biharmonic equation, and we seek a solution which gives a motion of uniform shear at large distances. For the chosen profile system, it is found that partial allowance for the neglected inertia terms can then be made, although for most problems of this type the first approximation is normally sufficient.

A FIRST APPROXIMATION FOR THE STREAM FUNCTION

In Fig. 1 the part of the z-plane lying above the boundary ABC can be represented conformally on the upper half of the ζ-plane by

$$-z = \frac{1}{\zeta} + \frac{1}{\zeta + i} + i; \tag{1}$$

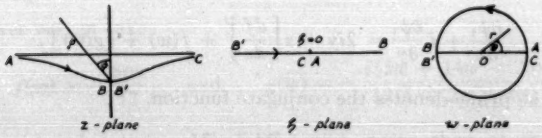

FIG. 1.

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the upper half of the ζ-plane can be represented conformally on the interior of the unit circle centre the origin in the w-plane by

$$\zeta = \frac{i(1-w)}{(1+w)}. \tag{2}$$

From Eqs. 1 and 2 we see that

$$z = -\frac{i(w+1)}{(w-1)} + \frac{i(w-1)}{2} \tag{3}$$

conformally transforms the area above the hollow into the interior of $|w| = 1$. At a point of the boundary ABC given by $w = e^{i\theta}$, we have

$$x = -\cot\frac{\theta}{2}\left(1 + \sin^2\frac{\theta}{2}\right), \qquad y = -\sin^2\frac{\theta}{2}, \qquad z = x + iy, \tag{4}$$

which gives the boundary profile in parametric form.

As a first approximation we neglect the inertia terms in the equations of motion, so that the stream function ψ_1 satisfies the biharmonic equation

$$\nabla^4\psi_1 = 0, \tag{5}$$

subject to the boundary condition $\psi_1 = \dfrac{\partial\psi_1}{\partial n} = 0$ on ABC, where $\partial/\partial n$ denotes differentiation with respect to distance measured normal to the boundary. The stream function must also be such as to give a motion of uniform shear at a great distance from B.

If we set

$$\psi_1 = y^2 + xf_1 + yf_2 + g_1, \tag{6}$$

where f_1, f_2 and g_1 are real functions such that

$$f_1 + if_2 = f(w), \qquad \frac{d}{dz}(g_1 + ig_2) = g(w),$$

then ψ_1 is biharmonic, and it is easy to see that

$$\frac{\partial\psi_1}{\partial x} + i\frac{\partial\psi_1}{\partial y} = 2iy + z\left[\frac{df}{dz}\right]' + f(w) + [g(w)]',$$

where the prime denotes the conjugate function.

Now on the boundary ABC $\dfrac{\partial\psi_1}{\partial x} = \dfrac{\partial\psi_1}{\partial y} = 0$, so that we require

functions f and g such that

$$2iy + z\left[\frac{df}{dz}\right]' + f(w) + [g(w)]' = 0, \tag{7}$$

when $wt = 1$, where t is the complex conjugate to w. When $wt = 1$, we have quite simply

$$2iy = \frac{i(w-1)^2}{2w},$$

and

$$\left[\frac{df}{dz}\right]' = \frac{2i(w-1)^2}{(5w^2 - 2w + 1)}\left[\frac{df}{dw}\right]',$$

so that Eq. 7 can be written

$$\frac{i}{2}\left(w - 2 + \frac{1}{w}\right) - \frac{(w-1)(w^2 - 4w - 1)}{(5w^2 - 2w + 1)}\left[\frac{df}{dw}\right]' + f(w) + [g(w)]' = 0. \tag{8}$$

By first taking

$$f(w) \equiv i \quad \text{and} \quad g(w) \equiv iw/2. \tag{9}$$

we satisfy

$$\frac{i}{2}\left(-2 + \frac{1}{w}\right) - \frac{(w-1)(w^2 - 4w - 1)}{(5w^2 - 2w + 1)}\left[\frac{df}{dw}\right]' + f(w) + [g(w)]' = 0,$$

and it thus remains to find further functions such that

$$\frac{iw}{2} - \frac{(w-1)(w^2 - 4w - 1)}{(5w^2 - 2w + 1)}\left[\frac{df}{dw}\right]' + f(w) + [g(w)]' = 0 \tag{10}$$

when $wt = 1$.

Since no singularities must be introduced into the region occupied by the liquid, we cannot choose obvious solutions $f(w) \equiv 0$, $g(w) \equiv \frac{i}{2w}$. However we can take

$$f(w) \equiv -\frac{5iw}{12} \quad \text{and} \quad g(w) \equiv -\frac{i(5w + 14w - 23)}{12(5 - 2w + w^2)}, \tag{11}$$

because the singularities of the function g are then at the points $w = 1 \pm 2i$ which are outside the region $|w| = 1$.

From Eqs. 9 and 11 we get the complete solution

$$f(w) \equiv i - \frac{5iw}{12}, \qquad g(w) \equiv \frac{iw}{2} + \frac{i(23 - 14w - 5w^2)}{12(5 - 2w + w^2)}. \tag{12}$$

By simple integration we get

$$G = g_1 + ig_2 = -\frac{w^2}{8} + \frac{5w}{24} + \frac{7}{6(w-1)} + B,$$

where B is an arbitrary constant. By taking $B = \frac{4}{3}$ the boundary becomes the streamline $\psi_1 = 0$, and the stream function is completely determined in the form

$$\psi_1 = \frac{(wt-1)^2}{(t-1)^2(w-1)^2}\left\{1 - \frac{1}{12}(t-1)(w-1)\right\}. \tag{13}$$

The factor $(wt - 1)^2$ clearly shows that the boundary conditions are satisfied.

We now have to verify that the expression for ψ_1 is of the correct form at a great distance from the hollow. At a large distance from B, we can set $w = 1 + \alpha$, where $|\alpha|$ is small, so that from Eq. 3 the significant part of z is $-\frac{2i}{\alpha} = r_1 e^{i\theta_1}$, say, where r_1 is large. By picking out the most important terms in the stream function we see that at a large distance from B

$$\psi_1 = y^2 + H(\theta_1),$$

where $H(\theta_1)$ is a finite function of θ_1 independent of r_1. This establishes the motion of uniform shear.

In order to find an expression for ψ_1 close to the point B, we note that z is approximately equal to $-i$ and w nearly equal to -1. Setting $z + i = \rho e^{i(\varphi + \pi/2)}$ (see Fig. 1) and $w = -1 + \gamma$, we can establish from Eq. 3 that $\rho e^{i\varphi} = \gamma + O(\gamma^2)$, so that we may take $\gamma = \rho e^{i\varphi}$ as a close approximation. It follows that

$$\psi_1 = \frac{\rho^2 \cos^2\phi}{6} + O(\rho^4), \tag{14}$$

and since ρ is small we can neglect the last term on the right hand side. It is to be noted that ψ_1 is then biharmonic and the boundary conditions are satisfied at the point B.

THE PRESSURE

The pressure p is readily found from the equations of motion, for p is the harmonic conjugate of $\mu\nabla^2\psi_1$, where μ is the coefficient of viscosity. The working is simplified by the substitutions $X = w - 1$, $Y = t - 1$, so that

$$\nabla^2\psi_1 = -\frac{4}{3} + \frac{40}{3}\,\mathrm{Re}\,\frac{1}{(X^2+4)}.$$

The pressure is thus given by

$$p = \frac{40}{3}\,\mu\,\mathrm{Im}\,\frac{1}{(X^2+4)} = \frac{40}{3}\,\mu\,\mathrm{Im}\left\{\frac{1}{w^2-2w+5}\right\}. \tag{15}$$

In particular, on the boundary $w = e^{i\theta}$

$$p = \frac{20\mu\sin\theta(1-\cos\theta)}{3(5\cos^2\theta - 6\cos\theta + 5)}. \tag{16}$$

PROFILE SYSTEM

The results we have obtained so far can be generalized to other hollows by considering the transformation

$$z = \frac{i(w+1)}{(1-w)} + \frac{ai(w-1)}{2}, \qquad a > 0 \tag{3-1}$$

instead of Eq. 3.

Figure 2 shows three such hollows: for example when $a = \frac{1}{2}$, the

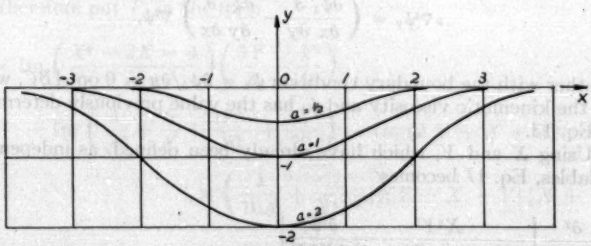

FIG. 2.

effective width and depth are respectively 4 and $\frac{1}{2}$ units; when $a = 1$ the dimensions are 6 and 1 units; and when $a = 2$ the dimensions are 8 and 2 units. Conversely, by considering effective width and depth we may approximate to any hollow by correct choice of a.

Using Eq. 3–1 we arrive at the stream function ψ_1 in the form

$$\psi_1 = (wt - 1)^2 \left\{ \frac{1}{(t-1)^2(w-1)^2} - \frac{a^2}{4(2+a)(t-1)(w-1)} \right\}. \quad (13\text{–}1)$$

An expression for the pressure can be similarly obtained.

ELASTIC ANALOGY

Apart from their significance in fluid-flow problems, the results so far obtained can be immediately used to find the tension in a semi-infinite elastic plane whose edge contains a projection and is traction free. This follows from the fact that the equations governing the slow steady flow of a viscous incompressible fluid are the same as those characterizing an equilibrium state of an incompressible elastic solid, if one simply replaces velocity and coefficient of viscosity by displacement and shear modulus.

SECOND APPROXIMATION

With so simple an expression for ψ_1, we can proceed a stage further, and make partial allowance for the effect of inertia. We consider only the case $a = 1$; for other values of a the calculation is basically the same. In order to save time and space much of the detailed working can be omitted, and so only the essential steps are given here.

For a second approximation to the stream function $\psi = \psi_1 + \psi_2$, we need to find a function ψ_2 satisfying the differential equation

$$\nu \nabla^4 \psi_2 = \left(\frac{\partial \psi_1}{\partial x}\frac{\partial}{\partial y} - \frac{\partial \psi_1}{\partial y}\frac{\partial}{\partial x} \right) \nabla^2 \psi, \quad (17)$$

together with the boundary condition $\psi_2 = \partial\psi_2/\partial n = 0$ on ABC, where ν is the kinematic viscosity and ψ_1 has the value previously determined by Eq. 13.

Using X and Y, which have already been defined, as independent variables, Eq. 17 becomes

$$4\nu \frac{\partial^2}{\partial X \partial Y} \left\{ \frac{X^2 Y^2}{(X^2+4)(Y^2+4)} \frac{\partial^2 \psi_2}{\partial X \partial Y} \right\}$$

$$= -\frac{5i}{36(X^2+4)^2} \left\{ -\frac{24X}{Y^3} + \frac{(X^2 - 24X - 24)}{Y^2} - (X+1)^2 \right\}$$

$$+ \frac{5i}{36(Y^2+4)^2} \left\{ -\frac{24Y}{X^3} + \frac{(Y^2 - 24Y - 24)}{X^2} - (Y+1)^2 \right\}. \quad (18)$$

A particular integral of Eq. 18 is obtained readily by direct integration in the form

$$\frac{72\nu\psi_2}{5} = \mathrm{Im}\left\{-X\left(\frac{2}{Y^3}+\frac{6}{Y^2}+\frac{3}{2Y}+\frac{Y^2}{8}\right)\right.$$
$$+\left(2X+\frac{7}{X^2}-\frac{3X^2}{16}\right)\log Y + 5\left(\frac{1}{X^2}+\frac{X^2}{16}\right)$$
$$\left.\times\left[\left(\frac{Y}{2}-\frac{2}{Y}\right)\tan^{-1}\frac{Y}{2} - \log\,(Y^2+4) + \log Y\right]\right\}. \quad (19)$$

To this particular integral we can add a complementary function which is biharmonic since we are finding solutions to the differential Eq. 17. Moreover, this biharmonic function must be chosen so that the boundary condition is satisfied, that is to say, our final expression for ψ_2 must contain a factor $(tw-1)^2$ or $(XY+X+Y)^2$ in terms of X, Y. Also the stream function must have no singularities inside $|w|=1$.

The expression (19) for ψ_2 contains terms in three distinct groups. We therefore start with the first group from the right hand side namely

$$T_1 = -\,\mathrm{Im}\left\{X\left(\frac{2}{Y^3}+\frac{6}{Y^2}+\frac{3}{2Y}+\frac{Y^2}{8}\right)\right\}.$$

It is easy to verify that $\mathrm{Im}\,(X/Y)$ is biharmonic, as also are terms of the type $\mathrm{Im}\left(\frac{X^2-2X-4}{X}\right)F(Y)$ where $F(Y)$ is any function of Y. We therefore put T_1 in the form

$$T_1 = \mathrm{Im}\left(\frac{X^2-2X-4}{X}\right)\left(\frac{3Y}{2}-\frac{Y^2}{40}\right)$$
$$-\,\mathrm{Im}\left(\frac{4}{X^2}+\frac{6}{X}+\frac{11X}{10}+\frac{3X^2}{20}\right) - \mathrm{Im}\,(XY+X+Y)^2$$
$$\times\left(\frac{1}{10X}+\frac{2}{X^3Y^3}\{XY+X+Y\}\{X+1\}\right),$$

where the first two sets of terms are biharmonic. Hence in our final expression for ψ_2, the group of terms T_1 can be replaced by

$$T_1' = -\,\mathrm{Im}\,(XY+X+Y)^2$$
$$\times\left(\frac{1}{10X}+\frac{2}{X^3Y^3}\{XY+X+Y\}\{X+1\}\right),$$

or in terms of w and t

$$T_1' = -\operatorname{Im}(tw-1)^2\left(\frac{1}{10(w-1)}+\frac{2w(tw-1)}{(t-1)^2(w-1)^2}\right), \quad (20)$$

which now satisfies the boundary condition, and contains no singularities inside $|w| = 1$.

In a similar manner we treat the other groups of terms from Eq. 19

$$T_2 = \operatorname{Im}\left(2X+\frac{7}{X^2}-\frac{3X^2}{16}\right)\log Y$$

$$T_3 = 5\operatorname{Im}\left(\frac{1}{X^2}+\frac{X^2}{16}\right)\left[\left(\frac{Y}{2}-\frac{2}{Y}\right)\tan^{-1}\frac{Y}{2}-\log(Y^2+4)+\log Y\right].$$

No details need be given here, but it is again stressed that singularities in the region occupied by the fluid must be avoided. It is found that T_2, T_3 may be replaced by

$$\begin{aligned} T_2' = \operatorname{Im}(tw-1)^2\Big[&-\frac{3}{16t^2}-\frac{3}{2t(w-1)}+\frac{7}{(t-1)^2(w-1)^2} \\ &-\frac{7}{2(t-1)(w-1)}+\frac{25t+3}{(w-1)(5t^2-2t+1)}\Big]\log(1-t) \\ &+\frac{3}{16}\operatorname{Im}(tw-1)^2\left[\frac{1}{w}-\frac{4}{5(w-1)}\right] \\ &-\operatorname{Im}\frac{(tw-1)^2}{(w-1)}\left[\frac{(25l-20k)t+3l+14k}{(5t^2-2t+1)}\right], \end{aligned} \quad (21)$$

$$\begin{aligned} T_3' = {}& 5\operatorname{Im}(tw-1)^2\left[\frac{1}{16t^2}+\frac{1}{(t-1)^2(w-1)^2}-\frac{1}{2t(t-1)(w-1)}\right] \\ &\times\left[\left(\frac{(t-1)}{2}-\frac{2}{(t-1)}\right)\tan^{-1}\frac{(t-1)}{2}-\log(t^2-2t+5)+\log(1-t)\right] \\ &+5\left(\frac{3k}{2}+\log 5\right)\operatorname{Im}\frac{(tw-1)^2}{(t-1)(w-1)}\left(\frac{t}{16w^2}-\frac{t}{16w}-\frac{1}{16w^2}+\frac{9}{16w}\right) \\ &-\frac{25k}{32}\operatorname{Im}(tw-1)^2\left(\frac{1}{w}-\frac{4}{5(w-1)}\right), \end{aligned} \quad (22)$$

where $l = \log 2 - \frac{1}{2}\log 5$ and $k = \tan^{-1}(\frac{1}{2})$.

The final expression for ψ_2 which now satisfies all necessary conditions is given by

$$\frac{72\nu}{5}\psi_2 = T_1' + T_2' + T_3' \tag{23}$$

where T_1', T_2', T_3' are given by Eqs. 20, 21 and 22.

As before, we can examine the behaviour of ψ_2 at large distances from the boundary, and using Eq. 23 obtain the asymptotic form

$$\frac{72\nu}{5}\psi_2 = 2y^2\{6(\theta_1 - \pi/2) - \sin 2\theta_1\}.$$

This expression takes equal and opposite values at the two points $(\pm x, y)$, as it ought, but it is not biharmonic. The stream function $\psi = \psi_1 + \psi_2$ is thus seen to tend to

$$\psi = y^2\left\{1 + \frac{5}{36\nu}[6(\theta_1 - \pi/2) - \sin 2\theta_1]\right\}. \tag{24}$$

From Eq. 24 it follows that the shearing condition at great distance is not exactly satisfied, but the discrepancy is very small when $\theta_1 \doteq \pi/2$, and this is the region of greatest interest being immediately above the hollow.

For points near to the point B we set $z + i = i\rho e^{i\phi}$ as before, and obtain from Eq. 23

$$\frac{72\nu}{5}\psi_2 = 4k_1\rho^3 \cos^2\phi \sin\phi$$

approximately where

$$k_1 = -\frac{7}{128} + \frac{5\pi}{16} + \frac{187}{64}\log 2 - \frac{37}{32}\log 5 - \frac{17}{8}\tan^{-1}(\tfrac{1}{2}).$$

The approximation here for ψ_2 is thus biharmonic, whereas the function ψ_2 itself is not. That this must be so can be seen without detailed calculation, for a first approximation of ψ_1 is of order ρ^2 as seen from Eq. 14, and hence a first approximation to the right hand side of Eq. 17 is of order ρ^0 and the corresponding particular integral for ψ_2 of order ρ^4. Any terms of lower order which occur must be part of the complementary function added in order to satisfy the boundary conditions, and therefore are biharmonic.

Finally, we remark that, if required, a second approximation to the pressure p can now be calculated without much difficulty. The expression for p is rather long and won't be given, because, for practical purposes, formulae 15 and 16 will usually be adequate.

Transistorized D-C. Amplifier.—Beckman Instruments, Inc., of Fullerton, Calif., is now offering a unique, completely transistorized d-c. amplifier, the Model WN, for use in situations where the need for high accuracy is combined with a need for absolute reliability.

The Model WN, a general-purpose, chopper-stabilized unit, was designed, according to Taylor Fletcher, manager of Beckman's Data and Control System's Department, to meet the high reliability standards demanded by chemical and petroleum companies for data handling equipment to be used in process-control. Amplifiers similar to the Model WN are used in Beckman 112 Systems now being built for a number of companies in the process industries.

The Model WN is actually a package of four separate amplifiers, each contributing to the precise amplification of the input signal. Two of these act as power supplies, producing an accurate plus or minus 25-volt potential. The power transformer in these units is completely isolated from earth ground, allowing the entire Model WN package to "float" relative to ground.

The signal itself passes through two amplifiers, one an a-c. unit that amplifies the chopped signal, the second a d-c. unit that takes the signal after it has been demodulated and filtered. A plug-in "can" of precision resistors permits rapid, foolproof change in net amplification ratio. The elimination of vacuum tubes in all four amplifiers means that the Model WN is completely free of microphonics, no matter what the service conditions.

Characteristics of the Model WN include: 2 μv drift in 24 hours at 25° C; .01 per cent linearity at d-c.; noise referred to input is only 3 μv p−p at 0−1 cps.; output is ±15 volts at 100 milliamp.; output impedance is .02 ohm. Power requirements are 105–125 volts, 60-cycle current.

Uranium Detector Available to Public.—The Radiac Company, Inc., New York City, has announced the release for unrestricted public use of the Model DS-911 Scintillacounter for oil and uranium prospecting. The Scintillacounter, a gun-type scintillation survey meter, was originally built on a United States Government contract to meet U. S. Atomic Energy Commission and Geological Survey requirements for a radioactivity detector to be used for aerial and mobile uranium surveys, and for prospecting on foot.

Although it was originally designed for the detection of deeply buried uranium ore bodies, the Scintillacounter can also be used for locating oil and gas deposits through the radioactivity anomalies technique (which outlines oil pools through a negative radiation count in an area of otherwise normal "background").

Its special features include an improved scintillation detection system with extremely high gamma sensitivity, a short resolving time and high counting rate. The newly designed high voltage circuit operates on standard low voltage batteries.

While the entire unit is water-resistant, the scintillation probe has been completely water-sealed to withstand submersion in water, in accordance with A.E.C. prospecting specifications.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS

REVERSAL OF PARITY LAW IN NUCLEAR PHYSICS

Recent low-temperature experiments at the National Bureau of Standards have demonstrated that the quantum mechanical law of conservation of parity does not hold in the beta decay of cobalt-60 nuclei.[1] This result, together with experiments on parity conservation in μ-meson decay at Columbia University's Nevis Cyclotron Laboratories, shatters a fundamental concept of nuclear physics that has been universally accepted for the past 30 years. It thus clears the way for a reconsideration of current physical theories with the possibility of new, far-reaching discoveries regarding the nature of matter and the universe.

In particular, removal of the restrictions imposed by parity conservation promises to bring order to the theoretical chaos now existing in regard to subatomic particles. It is generally held that the new "elementary" particles from proton accelerators are manifestations of the forces that bind the nucleus together. Thus, a better understanding of these particles may well lead to a more fundamental, unifying theory of the nature of matter and energy.

The beta-decay experiments were carried out by C. S. Wu of Columbia University and NBS staff members Ernest Ambler, Raymond W. Hayward, Dale D. Hoppes, and Ralph P. Hudson. The Bureau's low temperature laboratory was chosen for the experiments because of its previous experience in low-temperature alignment of atomic nuclei,[2] an essential feature of the beta-decay study.

Basically, parity conservation in quantum mechanics means that two physical systems, one of which is a mirror image of the other, must behave in identical fashion except for the mirror image effect. In other words, nature is symmetrical and makes no fundamental distinction between right- and left-handed rotations or between the opposite sides of a subatomic particle. Thus, for example, two similar radioactive particles spinning in opposite directions about the same axis should emit the same intensity of radioactivity in any given direction. Neither the right- nor the left-handed rotation should be favored by a greater intensity of emission so long as parity is conserved.

Since 1925 physicists have accepted the principle that parity is conserved in all types of interactions. During the past few years, however, phenomena have been described in high-energy physics that

[1] For further details, see "An Experimental Test of Parity Conservation in Beta Decay," by C. S. Wu, E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson, *Phys. Rev.*, (in press).

[2] "Low-temperature Alignment of Radioactive Nuclei," *NBS Tech. News Bul.*, Vol. 40, p. 49 (1956). See also, E. Ambler, R. P. Hudson, and G. M. Temmer, *Phys. Rev.*, Vol. 97, p. 1212 (1955) and Vol. 101, p. 1096 (1956).

could not be explained by existing theories. Large accelerators such as the Cosmotron and Bevatron have been producing a variety of subatomic particles whose behavior has defied systematic analysis. One such particle is the K meson, a short-lived particle which the proton beam ejects from atomic nuclei. Although K mesons are alike in all significant ways, some of them decay into three π mesons while others decay into only two π mesons. Mathematically this inconsistent behavior led to the suspicion that an explanation might be found in a violation of the principle of conservation of parity.

In the summer of 1956 T. D. Lee of Columbia University and C. N. Yang of the Institute for Advanced Study made a survey[3] of experimental information on the question of parity. They concluded that the evidence then existing neither supported nor refuted parity conservation in the so-called "weak interactions," such as emission of a beta particle or K-meson decay. They also proposed a number of experiments on beta decays and hyperon and meson decays that would provide the necessary evidence for or against parity conservation in weak interactions. One of the proposed experiments involved measuring the directional intensity of beta radiation from oriented cobalt-60 nuclei. At the suggestion of Professor Wu of Columbia, arrangements were made to carry out this experiment in the Bureau's low temperature laboratory.

Polarization of the nuclei was achieved by cooling a paramagnetic crystal containing cobalt-60 to within 0.01°C. of absolute zero, and subjecting it to a magnetic field. At this temperature the effects of thermal agitation are so small that atomic nuclei can line up in a given direction within the crystal lattice when a magnetic field is applied.

The magnetic polarity of the nucleus is determined by its direction of spin, and, under the influence of a magnetic field, most of the cobalt-60 nuclei align themselves so that their spin axes are parallel to the field.

As cobalt-60 is radioactive, its nuclei continuously emit beta and gamma rays. If parity is conserved in such interactions, then the intensity of the beta emission should be the same in either direction along the axis of spin. This, of course, was the critical question in the cobalt-60 experiments. It was resolved by measuring the intensity of beta emission in both these directions, i.e., along and against the field direction.

The cobalt-60 was located in a thin (0.002-in.) surface layer of a single crystal of cerium magnesium nitrate. The crystal was placed in an evacuated flask which in turn was immersed in liquid helium within a Dewar flask surrounded by liquid nitrogen. An inductance coil on the surface of the inner flask was used to measure the temperature of the crystal in terms of its magnetic susceptibility.

[3] T. D. Lee and C. N. Yang, *Phys. Rev.*, Vol. 104, p. 254 (1956).

A major experimental problem was the location of a radiation counter within the evacuated flask for detection of beta particles. This problem was solved by placing a thin anthracene crystal inside the chamber to serve as a scintillation counter. The anthracene crystal was located about 2 cm. above the cobalt-60 source. Scintillations caused by beta particles striking the crystal were transmitted through a glass window and a 4-ft. lucite tube acting as a light pipe to a photomultiplier at the top of the flask. The resulting pulses were counted on a 10-channel pulse-height analyzer.

In addition to the beta counter within the vacuum chamber, two sodium iodide gamma scintillation counters were used externally to measure the directional intensity of the more penetrating gamma radiation. In this way the investigators were able to determine the degree of polarization of the cobalt-60 nuclei. The two gamma counters were biased to accept only the pulses from the photopeaks in order to discriminate against pulses from Compton scattering.

Cooling to the low temperature necessary for nuclear alignment was accomplished by the process of adiabatic demagnetization using a magnetic field of about 23,000 oersteds. This process involved successive magnetization and demagnetization of the paramagnetic salt, cerium magnesium nitrate, which supported the cobalt-60 specimen. The heat produced by magnetization was removed by the boiling off of liquid helium in the surrounding dewar. The specimen was then thermally isolated and upon demagnetization the temperature fell to about 0.003° K.

A vertical solenoid was then raised around the lower end of the outer dewar to provide a magnetic field for polarization of the cobalt-60 nuclei. After the beta emission had been measured for this condition, the direction of the magnetic field was reversed, and the beta emission again measured for nuclei polarized in the opposite direction. It was found that the emission of beta particles is greater in the direction opposite to that of the nuclear spin. Thus, a spinning cobalt-60 nucleus has a beta emission distribution that is not the same as that of its mirror image. This result unequivocally demonstrates that parity is not conserved in the emission of beta particles by cobalt-60.

According to the theoretical studies of Professors Lee and Yang, the present experiment indicates not only that conservation of parity is violated in beta decay, but also that invariance under charge conjugation is violated.[4] This can be interpreted to mean that when the algebraic signs of all electric charges in a physical system are changed, the physical behavior of the system may not always remain unaltered. Most recently, Lee and Yang have developed a new theory for the neutrino as a result of the foregoing work.[5]

Further low-temperature studies of beta decay are proceeding at

[4] T. D. Lee, Reinhard Oehme, and C. N. Yang, *Phys. Rev.* (in press).

[5] T. D. Lee and C. N. Yang, *Phys. Rev.* (in press).

the Bureau. In this work it is hoped that the invariance of time reversal—changing the algebraic sign of time in time-space equations—can be decided.

IMPROVED HIGH-PRESSURE PACKING

Scientists of the National Bureau of Standards have successfully used polytetrafluoroethylene impregnated with 5 per cent by weight of molybdenum disulphide as a packing material in the high-pressure moving seal of differential-area pressure intensifiers.[1] The packing has served effectively at pressures approaching 200,000 psi. The use of this packing material was developed in the course of the NBS program, sponsored in part by the Army Ordnance Corps, to extend the range of high-pressure standards and improve techniques of pressure measurement.[2]

The impregnated polytetrafluoroethylene (Teflon) packing[3] was used with a mushroom plug and extrusion ring assembly similar to that described by Bridgman.[4] The ram of the intensifier pushes against a cylindrical ring of hardened steel, into the hollow of which the stem of the mushroom plug fits. Held in a framework of copper-beryllium extrusion rings, the packing is placed in the annular region between the hardened steel ring and the mushroom plug. In this position, the packing isolates the effectively zero pressure in the hollow of the steel ring from the high pressure at the annular surfaces where the packing touches the steel ring and the plug.

Intensifiers packed in this way were found to have from 10 to 15 per cent less over-all friction than similar intensifiers packed with leather. Since several other sources besides packing contribute to the over-all friction, the improvement in packing friction is fairly appreciable.

After more than 100 hours under load, with 18 pressure applications up to between 100,000 and 190,000 psi., one of the intensifiers was disassembled and examined. No signs of packing extrusion were found. In fact, the original tool marks were still visible on the surface of the plastic packing material that had been exposed to the rubbing action of the intensifier wall. The absence of extrusion is believed due either to the presence of the filler in the plastic or to the shape of the extrusion rings, or both.

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[1] "Impregnated Teflon as a Packing Material at 150,000 Pounds per Square Inch," by H. A. Bowman, J. L. Cross, D. P. Johnson, J. D. Hill, and J. S. Ives, *Rev. Sci. Instr.*, Vol. 27, p. 550 (1956).

[2] "High-pressure Standards," *NBS Tech. News Bul.*, Vol. 40, p. 96 (1956).

[3] The packing material was chosen after consultation with H. S. White, of the Bureau's engines and lubrication laboratory, who has used these materials in experiments on oil-free bearings.

[4] "The Physics of High Pressure," by P. W. Bridgman, pp. 34, 39, G. Bell and Sons, London, 1949; "The Design of High Pressure Plant and the Properties of Fluids at High Pressure," by D. M. Newitt, pp. 84, 86, Oxford University Press, 1940.

THE FRANKLIN INSTITUTE

MINUTES OF THE STATED MEETING

April 17, 1957

The Stated Monthly Meeting of The Franklin Institute was held on April 17, 1957, in the Lecture Hall. Mr. S. Wyman Rolph, President, called the meeting to order at 8:15 P.M. with approximately 165 members and guests in attendance.

The President stated that the minutes of the Stated Meeting of March 20 will be printed in the April issue of the JOURNAL and will be submitted for approval at the next Stated Meeting of the Institute on May 15, 1957.

Mr. Rolph announced three changes in the organization of The Franklin Institute. Mr. Hamilton Vaughan Bail, who has been Treasurer of the Institute for ten years and Secretary for the past year, resigned as Secretary as of March 31, 1957, and as Treasurer as of April 30, 1957. Mr. William F. Jackson, Jr. was elected Secretary at the March meeting of the Board of Managers to become effective April 1, 1957. Mr. Cecil M. Waterbury was elected Treasurer at the April meeting of the Board of Managers to become effective May 1, 1957.

Mr. Rolph extended a warm welcome to the officers and members of the Philadelphia Society for Promoting Agriculture, their seventeenth joint meeting with The Franklin Institute. He introduced Mr. J. Gordon Fetterman, President of this Society, who acted as Chairman for this joint meeting. For the officers and members of the Agricultural Society, Mr. Fetterman expressed thanks to the Institute for the privilege of holding joint meetings and commended the Institute for the work it is doing for the benefit of mankind.

Mr. Fetterman presented the Philadelphia Agricultural Award to Dr. William L. Myers, Dean, New York State College of Agriculture, Cornell University, Ithaca, New York. Dr. Myers acknowledged the award by expressing his appreciation for this honor. In his talk titled "Farm Programs and Food Prices" Dr. Myers emphasized the importance of the application of science to agriculture. He discussed broadly the economics and marketing applications to the farming industry and the need for promoting greater research and development projects to help this industry. Dr. Myers favored a free market for the farmers and the application of good management practices for the farming industry. This was a very interesting talk and was enthusiastically received by the audience with a rising vote of thanks.

The meeting was adjourned at 9:45 P.M.

WILLIAM F. JACKSON, JR.
Secretary

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, April 10, 1957.)

HALL OF THE COMMITTEE,
PHILADELPHIA, APRIL 10, 1957.

MR. JOHN H. NEHER in the Chair.

The following report was presented for final action:

No. 3320: Ballantine Medal.

This report recommended the award of a Stuart Ballantine Medal each to Leo Clifford Young and Robert Morris Page, of Washington, D. C., "In consideration of their outstanding achievement, the development of the first pulsed radar system in the United States."

D. S. FAHRNEY,
Secretary to Committee

MEMBERSHIP

ACTIVE MEMBERS ELECTED BY THE BOARD OF MANAGERS

March 20, 1957

ACTIVE FAMILY

Reynold H. Ackermann
Robert R. Ackley
Herbert J. Alburger
George H. Brown
Robert M. Joline
Edwin Keller
D. O. Landis
Lachlan Pitcairn
Morris V. Shelanski
Raymond K. Schwolow
Vincent L. Verdiani

ACTIVE

Joseph M. Allerdice
Vernon G. Ames
Francis J. Carey
Arthur F. Draper
Milton J. Freiwald
Louis Gerstley, Jr.
Mrs. Francis C. Grant
William F. Jackson, Jr.
Franklin R. Kay
W. A. LaLande, Jr.
Edward D. Marker
R. D. Morgan
Hyman B. Natkin
Robert W. Noble
Hyman Olken
Edward A. O'Neill
Adolph Oscar Schaefer
Norman J. Sendler
Robert K. Stanley
Mitchel Stern
Edgar C. vanDyke
Marshall D. Vaughters
John Sheldon Youtcheff
John Zimmerman

ACTIVE NON-RESIDENT

Michael K. Avedissian
John F. Lynn, Jr.
Robert G. Richardson
Charles G. Ritter
James Daniel West

NECROLOGY

M. M. Borden '29
Newell C. Bradley '40
Benjamin Ludlow '53
Irving D. Moultrop '34
Mrs. James S. Rogers '33
Harriet Sartain '43

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in this JOURNAL within the next few months:

COULSON, THOMAS: Prominent Members of The Franklin Institute. No. 5. Alexander Dallas Bache, 1806–1867.

KLEIN, BERTRAM: Shear Buckling of Simply Supported Plates in Planform

HODGE, P. G., JR. AND JOHN PAPA: Rotating Disks with No Plane of Symmetry

GARDNER, GEORGE S: Researches on Corrosion and Inhibition. Adsorption, Inhibition, and the Langmuir Equation

GOLD, LOUIS: Note on the Relativistic Harmonic Oscillator

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor. 402 items have been added the past month.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open from 9 A.M. until 5 P.M. on Mondays, Tuesdays, Thursdays, Fridays, and Saturdays; and 2 P.M. until 10 P.M. on Wednesdays. Beginning June 1, Saturday hours will be from 9 A.M. until Noon.

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THE FRANKLIN INSTITUTE LABORATORIES FOR
RESEARCH AND DEVELOPMENT
NICOL H. SMITH, DIRECTOR

ELECTRIC ANALOG STUDY OF HYDROSTATIC BEARINGS*

BY

ALFRED M. LOEB

Hydrodynamic bearings are those bearings in which the fluid pressure needed to support the load is generated within the bearing itself by the relative motion of the stationary and moving parts. The load carrying capacity of bearings of this type is therefore dependent on the relative shaft speed. In applications where loads are high and speeds low, hydrodynamic bearings are limited in their usefulness. This limitation may be overcome by supplying pressurized lubricant, from an external source, to the load zone of the bearing. These supply pressures are usually far in excess of those normally used to supply lubricant to bearings. This type of bearing is said to be hydrostatically lubricated. In many cases, where high starting loads exist, the load is hydrostatically supported until the unit is up to speed, at which time the external pressure is removed and the load is then supported hydrodynamically. In other instances where the speeds are low, the hydrostatic pressure is applied continuously.

FIG. 1. Equipment for electric analog method, showing a conducting-paper model of one-quarter of the bearing shown in Fig. 2.

* Excerpt from a paper submitted for presentation at the ASME-ASLE Lubrication Conference, October 1957 in Toronto, Ontario.

In many applications of hydrostatic lubrication to journal and thrust bearings, determination of flow and load carrying capacity is at best an approximation. Anything but the simplest shape is difficult to analyze. The method of analysis employed in these experiments substitutes an analogous two-dimensional electric field for the hydraulic film in the bearing. Briefly, the method is as follows:

On Teledeltos paper, which has approximately equal electrical resistance in all directions, the boundaries and pressure sources of the bearing being investigated are painted with silver paint. By properly applying an electric potential (analogous to the hydraulic potential) to the silver electrodes, lines of equal potential may be plotted by

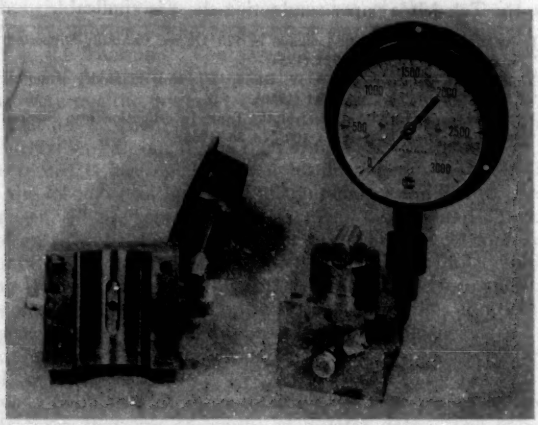

FIG. 2. Test bearings built to verify the electric analog analysis.

means of a suitable potentiometer arrangement. By numerical integration, the load carrying capacity may be determined. The flow requirements are established by measuring the electrical resistance and converting this to the so-called hydraulic resistance. Experimental verification of the journal bearing indicates that this method may be employed with considerable accuracy.

The use of electric analogs for the solution of flow and load carrying capacity of bearings is not new. This technique was first employed by Kingsbury[1] who gave the solution for a number of thrust and journal bearings, and showed close agreement with analytical solutions. This

[1] A. KINGSBURY, "On Problems in the Theory of Fluid-Film Lubrication, with an Experimental Solution," *Trans. ASME*, Vol. 53, p. 59 (1931).

work was continued by Needs[2] who investigated the effect of side leakage in a 120-degree centrally supported journal bearing. Morgan, Muskat and Reed[3] investigated the effect of circumferential grooves and supply sources within journal bearings. Kettleborough[4] investigated the stepped thrust bearing. These authors employed the electrolytic tank in their work, because the problems they investigated required a varying film thickness and therefore a varying depth in the electrolytic tank. In addition to this, it was necessary for them to supply a distributed current to the surface of the conducting sheet to complete the analogy. With hydrostatic bearings, since the film thickness is constant and the relative velocity of the bearing surface is zero, the need for a distributed current and a varying film thickness is eliminated. For these reasons, the Teledeltos paper conducting sheet is applicable.

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[2] S. J. NEEDS, "Effects of Side-Leakage in 120 Degree Centrally-Supported Journal Bearing," *Trans. ASME*, Vol. 56, p. 721 (1934).

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NOTES FROM THE FRANKLIN INSTITUTE MUSEUM

A. C. CARLTON, DIRECTOR

P. R. NO. 3

With so many specialists interested in the Museum there is not much chance for a mystery to exist in it, but there is one (and quite a large one from the physical viewpoint), which we should like to have solved. It is a locomotive, and it is named the "People's Railway No. 3."

Although undoubtedly in the possession of that railroad at one time, the locomotive is the property of the Reading Railroad which lends it to the Museum. The name of the builder is not known, nor can it be said definitely who were the original owners. The People's Railway were not the original owners. They acquired "No. 3" sometime about 1872, when they were opening a line between Pottsville and Minersville, Pa., but it was then some thirty years old.

The record is incomplete, but it would appear that "No. 3" was one of the locomotives built in Philadelphia by the firm of Eastwick & Harrison. This firm built the famous "Gowan and Marx" (the name of a British banking firm, by the way) which, at its trials in 1840, demonstrated a remarkable hauling power. The Philadelphia and Reading Railroad were so highly satisfied with the performance of their new locomotive that the road ordered the construction of others of the same general style. A careful comparison of the dimensions of "No. 3" with contemporary drawings leads to the assumption that it was one of the locomotives ordered.

Anyone familiar with the locomotives of that period will observe some features that are obviously of a later date. The cab is not original, but resembles a model that was introduced by the Reading shops about 1852, and is a contributory shred of evidence pointing to ownership by that road before it passed into the possession of the People's Railway. The sandbox is operated from the cab, a feature that was not introduced until about 1846. The truck wheels bear the name of A. Whitney & Sons, who did not open their wheel foundry until 1846. The smokestack is almost certainly not in its original form. "No. 3" in its pristine condition was a wood-burner. The firebox is of the Bury type, which was in common use until 1850. The old valve gear has been replaced by a double eccentric motion.

These, and other details, help to make "No. 3" a challenge to the student of old locomotives, but there seems enough evidence to justify the belief the locomotive was built for the Reading RR and transferred to the People's Railway, later to be returned to its original owner.

The steam locomotive is vanishing from the railroad scene. As each old specimen reaches the end of its usefulness it retains a monetary value not conducive to preservation. The scrap pile is waiting to receive it. This adds to the value of those locomotives which have escaped destruction. The Museum is fortunate to have secured three excellent examples of this type of engineering which serve to keep us instructed upon the form of transportation that made possible the expansion of the continent.

In the days of the present generation's childhood the passing locomotive was an occasion for awe, with its ruddy glow illuminating the interior of the cab, its shrill whistle, and the long plume of white smoke that marked its progress. Perhaps future generations will be the losers by knowing nothing of this reminiscence of the past.

Very few museums can allot the necessary space to demonstrate the story of railroading with original locomotives. We have found it necessary to complete the story with models. In this respect, too, the Institute Museum has been fortunate in obtaining some splendid examples of model-making which help to make the story of the steam locomotive more complete.

But the story of the American railroads does not end with the steam locomotive. Newer developments have improved efficiency, electric and diesel powered locomotives have taken the place of steam, and the Museum is not neglecting to display the contemporary developments. Since the hobby of model railroading has shown great expansion there is a revival of interest in the historical development. The construction of the large model railroad by the Museum staff is attracting widespread attention among both young and old. Visitors are watching progress with keen interest—but they turn and cast admiring eyes upon the three large superannuated locomotives that share the room. That, they seem to say, is where it all began.

Seminar on Tumor Immunity and Tumor Specific Substances.—DR. ARTHUR E. BOGDEN of the Wistar Institute of Anatomy and Biology, Philadelphia, Pennsylvania, addressed the Seminar of the Biochemical Research Foundation on Thursday afternoon, January thirty-first.

An immunological approach to the study of distinctive tumor constituents and the possibility of an immune response in cancer has long been burdened by Ehrlich's dramatic appellation "horror autotoxicus" to the possibility of spontaneous autoimmunization. The immune process is clearly one of the body's defense mechanisms against infection. Ideally, therefore, the body should respond immunologically only to the antigens of pathogenic microorganisms, viruses, and parasites.

In general, however, neoplasia cannot be regarded as an infectious disease due to a certain pathogenic micro-organism, but as originally normal cells whose autonomous cellular multiplication is uncontrolled by the usual regulatory mechanisms.

It is generally believed that the exciting stimulus which produces a malignant neoplasm creates some hereditary transmissible change in the "normal" cell which is passed through successive generations by cell division and which may contribute to the driving force for reduplication. For example, the continued presence of such agents as transient ionizing radiations or chemical carcinogens that undergo successive dilution in each cell division is clearly not required for the perpetuation of the cancers that these agents induce. Therefore, under circumstances of cancer pathogenesis that generally pertain, it is necessary to consider the presence of a hereditary transmissible factor.

While studies on higher organisms have revealed a great deal about the mechanics of gene-action, tremendous progress in the understanding of gene action in more meaningful biochemical terms has been made possible when microorganisms became the tools of genetic studies. That genes are the basic determinants of the synthetic chemical activities of the cell and that they are capable of controlling specific enzymatic processes has been shown by studying the effects of mutations affecting the cells' ability to synthesize some of their growth requirements.

However, phenotypic manifestations of gene mutation are not confined to cellular metabolism. From the transformation experiments of Sonneborn and his associates with the Paramecium, it can be concluded that nuclear genes determine which antigenic types may arise in a given strain (1). Beale, also using *Paramecium aurelia*, Variety 1, has shown that, in a homozygous stock, one of three immunogenetic loci comes to full expression at a given moment, and in heterozygotes

two antigens are regularly formed by the same cell (2). In certain bacteria antigenic changes occur spontaneously at very high rates. This is particularly true of changes in flagellar antigen, i.e., phase variation, in *Salmonella typhimurium*. Although "smooth" and "rough" are applied to colonial morphology, it is recognized that such changes are associated with changes in antigenic characteristics.

As postulated by Braun (3), we may assume that a loss or change of an antigenic component represents a process similar to that of a biochemical mutation; and just as an auxotrophic mutant may revert to a prototrophic condition, the loss of, or change in, an antigenic component may be restored by mutation to the original condition, a situation amply documented by observations and demonstrated by studies of the H antigen of *Salmonella typhimurium* by Stocker (4). Thus studies of the past few years have provided ample evidence that most changes in the antigenic characteristics of micro-organisms are mutational in origin.

Can we assume, therefore, in view of the fact that most of the mutagens used to increase normal mutation rates in micro-organisms, act also as carcinogens in the production of neoplasms, and in view of the mounting evidence that antigenic variations and specificity are genically controlled, that no change occurs in the antigenic mosaic of a cell when it mutates from normal to malignant?

The concept of "Immuno-catalysis" by Sevag (5), "that enzymatic groups, or catalytic centers, may be in actuality the determinants of the antigenic specificities of protein molecules," would lend additional weight to the immunologic approach to oncology and the study of distinctive tumor constituents.

The critical role of antigen-antibody reactions in the pathogenesis of certain types of glomerulonephritis and in the development of various lesions of the heart, blood vessels, skin, and joints in allergic, hypersensitive, and other pathologic states has been indited by the extensive work of the Caveltis, of Witebsky, and other investigators in recent years (6, 7, 8, 9, 10, 11, 12, 13). As stated by Hauschka (14), "Neoplastic antigens proper may indeed be ephemeral, since a spontaneous tumor is not a foreign tissue and cannot evolve as measurable a response as the iso-antibodies elicited by foreign grafts: yet, the reality of auto-immunization in health and disease argues against premature abandonment of the search for them."

An increasing awareness of the importance of an inbred animal in tissue transplantation studies, and the use of isogenic tumor-host systems for the elaboration of immune phenomena in cancer, has resulted from the studies of Immunogeneticists such as Snell, Kaliss, *et al.*

Reviews by Bittner (15) and Little (16) contain extensive evidence supporting the generally accepted principle that successful transplantation of both normal and neoplastic tissues is dependent upon the

presence of dominant genes simultaneously in graft and host. If a tumor graft carries one or more dominant loci functioning in the elaboration of cellular antigens and the host has inactive or immunologically different alleles of these genes (termed histocompatibility genes), the implant evokes antibodies against itself and usually regresses. If, on the other hand, the pertinent antigenic alleles are dominant in both tumor and recipient, no immune reaction ensues, and the host succumbs. The designation "dominant" versus "recessive" is perhaps an oversimplification (17). Both alleles could produce specific antigens, as in the case of the blood groups which, in many respects, can be considered as models for the immunogenetics of tumor transplantation. As concluded by Hauschka (14), "Inbred strains of animals and tumors which have originated in these pure lines are the best available material for immunogenetic tests."

A desire to simulate, as closely as possible in a large population of laboratory animals, the homogeneity that appears to exist between a spontaneous tumor and its host, and cognizance of the histocompatibility pitfalls in the use of transplanted tumors, led to the use of two inbred albino rat strains and tumors which originated spontaneously or that were induced by methylcholanthrene in these strains.

The P. A. strain, originally inbred (brother × sister matings) for 135 generations by Doctor Helen Dean King as the King Albino, was redesignated and inbreeding continued in the Aptekman Colony of the Wistar Institute since 1945.

The Lewis strain, originally a Wistar rat, has been inbred similarly for over ten years by Dr. Margaret Reed Lewis in her colony at the Wistar Institute.

It is realized that a strictly inbred strain of animals is essential in the immunologic approach to cancer, especially in the search for tumor specific substances, and that the basic value of such studies depends upon the maintenance of homogeneity of the transplantable tumors and their rat hosts. Therefore, a spontaneous or an induced tumor is transplanted only in the offspring (single trunk breeding) of the litter in which the tumor originated, and this tumor is never transplanted or used experimentally in the offspring of any other tumor subline.

Thus it is felt that such relatively strong antigens as those specific for species, strain and even subline differences, as well as histocompatibility factors have been at least minimized in these studies.

The study of tumor-immunity being carried out at the Wistar Institute was first sparked by Doctor Margaret Reed Lewis. She brought together Doctor Paul Myron Aptekman, a biochemist and Curator of the Wistar Museum, and the late Doctor Helen King, a geneticist whose inbred rat strain, the King Albino, was used for these studies.

The first publication resulting from this collaboration was on "A

Sarcomas-Producing Factor Extractable from Transplanted Rat Fibrosarcoma," published in *Cancer Research* in 1943 (18). This study crystallized an interesting possibility. If substances, irritating enough to be carcinogenic, could be extracted from malignant tissues, perhaps such tissues contained a substance or substances that would be specifically toxic enough to be oncolytic. The biochemical problem of extraction of such a substance fell to Doctor Paul M. Aptekman.

One of the steps in fat extraction by means of benzol, was the preliminary extraction of the minced tumor tissue with an equal volume of 95 per cent ethanol. Using this previously discarded fraction as the basic extract Dr. Aptekman developed a "concentrate" upon which the following studies on tumor-immunity and tumor peculiar substances are based (19). The primary extraction of minced tissue is effected with approximately 52 per cent ethanol. In the final, crude extract, however, the ethanol concentration is roughly 82 per cent. Therefore, the active oncolytic and immunizing factor or factors, is soluble in 82 per cent ethanol.

Rats of the inbred albino strain were selected for use in the studies on the production of immunity, since in previous studies it had been found that grafts of the sarcomata native in these rats had grown in every one (more than 5000) of the rats engrafted. No spontaneous regressions of any of the transplanted tumors were observed.

Rats of one litter were implanted on the right side of the body with approximately equal-sized grafts. When the tumor grafts had grown for 4–7 days and had attained a size of about 20 × 8 × 5 mm., part of them were injected intratumorally with one of the tumor concentrates, two were left untreated to serve as controls, and the others were treated with a control solution, since it was possible that solutions other than those containing extracts of tumors might destroy the growth of tumor cells. In all 45 rats, that were litter mates of those injected with tumor concentrates, were injected with one of the following control solutions: physiological salt solution, normal rat tissue extracted in saline solution, normal rat tissue, and also beef muscle extracted in ethanol and concentrated to 22–28 per cent alcohol and different strengths of ethanol (0.2–50 per cent). The results were as follows: fifty-six of the 58 tumors treated with a tumor concentrate were destroyed.

The damaged tissue in treated rats sloughed off, leaving an open sore that soon healed. Hair grew over the denuded area and eventually only a small scar showed the site of the tumor that had undergone oncolysis. Twenty-four of these healed rats were kept under observation for 8–12 months; the remaining 32 rats were implanted on the other side with a graft of the same type of tumor that had been destroyed in order to determine whether they were resistant to a second graft of this tumor. At the same time normal rats of the same age

and strain were implanted with a graft of the same tumor used in the treated rats. The grafts grew in every one of the untreated rats but failed to grow in 25 of the 32 (78.1 per cent) rats whose tumors had been destroyed by injections of the tumor concentrates.

In contrast to these results, tumors treated with the control solutions mentioned earlier, such as saline, saline extracts of normal rat tissue, ethanol extracts of normal tissues prepared in the same manner as the tumor concentrates, and various strengths of ethanol (up to 50 per cent), were not destroyed.

Some of the tumors treated with the control solutions, however, did become necrotic and opened to the surface; nevertheless, the malignant cells at the periphery continued to multiply toward the body wall, and the tumors increased in size until they brought about the death of the host, regardless of the control solution injected.

The reason for stressing this particular experiment is not because of any possible clinical applications, but because of its effectiveness as an immunizing procedure.

At the time of the publication of this study in 1945, these were the first experiments in which the injection into tumors of a substance such as the "tumor concentrate" brought about oncolysis of a tumor that arose in rats of its own inbred strain, and was followed by the establishment of tumor immunity in the majority of the treated rats (19).

Many investigators interested in immunity in relation to bacterial disease find that a certain period of time is needed for the elaboration of antibodies following an immunizing procedure.

In the earlier studies on the development of tumor immunity following oncolysis induced by intratumoral injections of a tumor extract, a period of three weeks was allowed to elapse between the last treatment and the challenge of the resistance of the animals.

Three weeks had been chosen to allow healing of the treated, denuded areas even though challenge grafts were inoculated on the opposite, untreated side.

Since the percentage of treated animals in which resistance to further grafts of their native tumor had been induced was quite high, it was decided to determine how soon after beginning such an immunizing procedure would the treated animal resist a challenge graft (20).

The experimental setup was a simple one. The inbred King A and Lewis Strain rats were inoculated on the right side with viable tumor tissue, and intratumoral treatments were begun seven days later. Treatments were scheduled at 2 day intervals.

Beginning with the 3rd treatment and every treatment thereafter groups of animals were inoculated with a challenge graft of tumor on the left, or untreated side.

Sixty-three of the treated rats were tested for resistance after the sixth intratumoral injection of tumor extract. Every one of them had,

by this time, developed sufficient tumor resistance to protect it from the growth of another graft of tumor tissue. This challenge graft failed to grow in these rats even though the tumor undergoing oncolysis on the right side still required further treatment in order to complete its destruction. There was a 12 days lapse of time between the first intratumoral treatment and inoculation of the challenge graft.

The percentage of resistant animals declined with the fewer number of treatments. None were resistant after three treatments and 40 per cent were resistant after four.

An immunizing graft of viable tumor tissue inoculated 1–2 months later also failed to grow in the resistant animal.

The results of these studies raised the question concerning the source of the immunizing agent. Does the development of resistance to the growth of transplanted sarcoma that takes place in cured rats result from absorption of inactivated tumor tissue, from the absorption of the alcoholic tumor extract, or possibly from some combination of serum or tissue and extract?

Studies were undertaken in an effort to learn the relation of the tumor extract to subsequent immunity; that is, whether it is possible to induce tumor-resistance in normal young susceptible rats by subcutaneous injections of an alcoholic extract of sarcomata prepared in the same manner as that used for intratumoral injections.

The results show that the response of the rats of different litters to this type of immunization varied from 0 to 100 per cent. Forty-seven of the 94 rats treated subcutaneously proved to be resistant to tumor growth, while 47 of them were found to be susceptible, in spite of having received the same number of injections of tumor extract. These results indicate that 50 per cent of the rats of the King A strain (19 litters selected at random) developed tumor resistance from injections of the alcoholic extract of sarcomata.

The 47 resistant rats that were kept under observation for several months failed to develop tumors following any of the several implantations of large tumor grafts that they received during this time. Each of the 100 normal young rats and the 30 older rats engrafted with portions of the sarcomata used to engraft the treated rats developed a tumor, showing that the tumor tissue used was viable and that the rats of this strain and age were susceptible to the growth of the tumors used (21).

Thinking along classical immunologic lines, it would appear that this tumor extract contained tumor specific substances which were antigenic in their own right, or as haptens conjugated to normal or denatured body proteins. The fact that these studies had been kept as rigidly as possible within an isogenetic system tended to support this.

The fact also, that intratumoral treatment resulted in a more effective immunization than subcutaneous treatment, was classically

true to form. An animal that undergoes an infectious process, though attenuated, and recovers from the disease has a greater degree of resistance than an animal actively immunized by means of a vaccine only.

Since immunization by subcutaneous treatment was the most practical and acceptable method of immunization, improving the effectiveness of the tumor extract as a "vaccine" was the next logical goal.

The tumor concentrate was toxic, not only upon intratumoral treatment, but also in the areas of subcutaneous injection. Therefore, removing the toxicity without destroying the desired antigenicity was the first step. Separation of most of the toxic fraction was made possible by dialysis.

During adsorption studies of the toxic dialysate in which human erythrocytes of blood groups O and AB (Rh positive) were being used, it was noticed that serum from both normal and tumor-immune rats had the ability to agglutinate these human erythrocytes, whereas serum from tumor-bearing animals did not.

This was so striking a phenomenon that a number of normal animals, and tumor-bearing animals with a variety of tumors, were tested. There was a consistent absence of hemagglutinins in the sera of 38 mature, tumor-bearing rats of the P. A. and Lewis strains. Each rat tested bore one of eight different transplantable tumors, or a primary induced tumor.

Studies on the influence of tumor inoculation and subsequent growth on these natural heterohemagglutinins revealed the following: Fifteen hours after implantation of malignant tissue in rats of either the P. A. or Lewis strain, an appreciable drop in the heteroagglutinin titers for cells of both O and AB blood groups occurred. The maximum immediate drop was reached in 48 hours. Thereafter, there was a rise in titers which reached a peak in 6–9 days. This rise, however, failed to reach the normal, pretumor implantation level. After 9 days there was a gradual and increasing drop in titer until the hemagglutinins disappeared entirely after 21 days.

Correlating tumor size with hemagglutinin titer in rats with progressively growing tumors, revealed that an inverse relationship existed between hemagglutinin titer and tumor growth (**22**).

This "disappearance phenomenon" was manifest in the homologous, as well as in the heterologous systems in three strains of rats (P. A., Lewis, and Wistar) having natural heteroagglutinins for erythrocytes of human blood groups AB and O. Since such a drop in humoral antibody titer following tumor implantation is suggestive of a "negative phase," it seemed advisable to determine whether the hemagglutinin reacting with the AB cell was reacting with the A component, the B component, or an antigen peculiar to the group AB cell, and whether

the anti-O cell agglutinin was a specific antibody or in some way serologically related to the group AB cell.

Studies characterizing these natural heteroagglutinins for human red cells present in the serum of normal, mature rats, revealed that the natural hemagglutinin peculiar to P. A. strain serum, reacting with human blood group AB and A cells, was serologically related to the human blood group A specific substance.

None of the natural heteroagglutinins reacting with blood group B and O cells were serologically related to either the blood group B or the O (H) specific substances.

The antibody causing agglutination in low titer, of group B and O cells, was shown to be reacting with a substance common to cells of all the four blood groups (23).

Since these heterohemagglutinins are normally present in the humoral system of the rat, a serologic relationship between these antibodies and antigens present in normal rat tissues would not be indicated. Furthermore, a "negative phase" in the humoral antibody titer is suggestive of an *in vivo* adsorption of antibodies by the injection of a serologically related antigen. In view of the fact, that an isologous relationship exists between the transplantable tumors studied and their rat hosts, serologic reactivity between extracts of these tumors and the characterized hemagglutinins would indicate the presence of a tumor specific substance.

Although the choice of the method of tissue extraction is dictated by the type of constituent sought, a survey of the literature shows that most of the immunological studies of neoplasia have been made with water or saline extracts. Strain and species lines were then crossed by means of such classical antibody producers as guinea pigs and rabbits as the recipients of rat or mouse tumor extracts. Even the exhaustive studies of Kidd, Friedewald and MacKenzie (24, 25, 26), with saline extracts of the Brown-Pearce carcinoma in rabbits, are subject to the question of histocompatibility, since this tumor arose in a hybrid animal, was transplanted in mixed stock for 20 years, and complement fixation by the specific antibody is predictable only for a restricted number of hybrid genotypes (14). Furthermore, it is felt that whatever antigenic differences may exist between normal and malignant tissues, in an isologous system, are very subtle so that serious consideration must be given to the possibility of masking of these subtle differences by the mass of antigens extractable from tissues by water and saline.

Since it was demonstrated by Aptekman and Lewis that ethanol extracts of rat sarcomata possessed immunizing as well as oncolytic properties, tissue extraction by their method was decided upon for the following studies (27).

Concentrates were prepared from the regular ethanol extract of

normal and malignant tissues, as well as the dialyzed and dialysate fractions of the concentrates of such tissues. It was found that an A and an X substance are present in the tumor concentrates, substances which appear to be common to all of the regular *tumor* concentrates studied; that the X substance, only, is dialyzable through a cellophane membrane, and is common to the regular concentrates of both normal and malignant tissues; and that lack of reactivity of normal tissue concentrates with normal serum in block titrations may be primarily due to the low molecular weight of the X substance.

This latter postulation is supported by the observations of Miller and Seibert (**28, 29**), in their biochemical analyses of the ethanol extracts of normal and malignant tissues prepared according to the procedure of Aptekman and Lewis. Electrophoretic and quantitative ninhydrin determinations of the composition of the non-dialyzable fraction of normal tissue extracts indicated that the protein or polypeptide molecules which were present were very small, although of sufficient size as not to dialyze through a colloidin membrane. Miller also found that the ethanol extract of normal rat muscle showed no antigenic activity when injected into rabbits, in contrast to the antibody response elicited by similar extracts of a rat lymphoma and sarcoma.

It is interesting, however, that the present study reveals an ethanol extractable substance common to both normal muscle and the tumors studied, since such transplantable tumors do contain normal, supportive, connective tissue stroma. The fact that the concentrate of normal muscle contained molecules too small to produce a visible reaction with normal serum *in vitro* does not minimize the possibility that this substance may be responsible for the cross reactivity observed by others in attempts to differentiate normal and neoplastic tissues by complement fixation techniques in humans and in other animal species.

In (cross) precipitin reaction studies by Miller (**28**) cross reactivity was observed between the non-dialyzable sarcoma and lymphoma proteins. Her studies by means of paper electrophoresis also suggested a relationship between the two tumors that was not evident with normal muscle. Witebsky and Morelli (**30**), using the complement fixation technique, found cross reactions between alcoholic extracts of human sarcoma and carcinoma tissues. Mann and Welker (**31, 32, 33, 34**) also found cross reactions between various human tumors.

The similarity of the reaction patterns observed with the tumor concentrates in block titrations, as well as in hemagglutination-inhibition tests, indicates that all the tumors studied contain a serologically active substance that inhibits the reactivity of the anti-A hemagglutinin. The characterization of this common substance peculiar to malignant tissue has been made only by means of the normal hetero-antibody system; therefore, the role that this "A"

substance plays in the oncolytic and immunizing activity observed by Aptekman and Lewis with similar tumor concentrates is unknown. Further fractionation studies will be made to ascertain its role, in an isologous system, since this substance appears to be peculiar to tumor tissue.

Thus it would appear that the substances responsible for the negative phase in the humoral antibody titer, that follows 24 to 48 hours after subcutaneous inoculation of viable tumor tissue, is the ethanol extractable "A" substance peculiar to neoplastic tissues and reacting with the anti-A hemagglutinin; and a non-specific "X" substance reacting with, or inhibiting the action of, the anti-X hemagglutinin reactive with erythrocytes of all the four blood groups (27).

SUMMARY

This seminar has been concerned with the presentation of a rationale for the existence of tumor specific substances and an immune response in cancer.

The importance of the inbred animal and an isogenic tumor-host relationship has been stressed.

Experiments showing that a tumor-resistant state can be induced in such an isologous system have also been presented.

Certain natural heterohemagglutinins present in normal rat serum and involved in a "negative phase" following tumor implantation, have been used to serologically characterize an ethanol extractable substance common to both normal and malignant rat tissues, and a substance which appears to be peculiar to the several rat neoplasia studied.

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BOOK REVIEWS

LEARNING AND INSTINCT IN ANIMALS, by W. H. Thorpe. 493 pages, illustrations, 6 × 9 in. Cambridge, Harvard University Press, 1956. Price, $10.00.

Professor Thorpe has written a book on the most central topic of modern psychology if the number of publications in the American literature is taken as a criterion of centrality. The sheer bulk of material on the topic of learning provided a challenge yet to be met by any other author successfully but Thorpe has taken on even more than the traditional behavioral studies and theories. He, in true European fashion, has attempted a most ambitious task of synthesis between the American behavioral-experimental data, the increasing European ethological observations, and current neuralphysiological models. This is a formidable task. In some ways he has provided a very valuable book and in others he falls far short of his goal, which is not at all surprising in view of the chaotic state of modern learning theory.

First of all let us outline how Thorpe goes about his task. The book is divided into three parts. Part I is a quite provocative discussion of the general concepts which have plagued learning theorists from the beginning. These are directiveness and purposiveness, and the nature of drive and its place in the theory of instinct. Thorpe concludes in line with his general cognitive bias that basic to learning is a general exploratory drive state superior to instinct but closely associated with innate response mechanisms.

Part II consists of 110 pages which treat the topics of habituation, associative learning (conditioning and trial-and-error), latent learning and insight learning, insight and perception, and concludes with a discussion of the neurological mechanisms which have been postulated to explain learning. Here again, as the choice of topics indicates, the author's general commitment to a cognitive type theory stands out. This section is definitely not for the uninitiated. The author cites a confusing array of evidence often rather uncritically.

The third part consists of about 250 pages of fascinating material on ethological and experimental studies of several species from Protozoa to Mammals with special attention to insects and birds. This section provides the real value of this book. It makes it the only truly comparative work on learning to appear in recent times and it is not surprising that its author is not American trained.

Thorpe then has attempted an ambitious task and has done a very creditable job. The book will be most stimulating for the advanced student of learning and the researcher will find the truly comprehensive bibliography in the ethological area very valuable. Both will be frustrated by the sometimes uncritical acceptance of poorly done studies. Thorpe throws out enough new and good ideas to offset this frustration if one reads selectively and critically.

DONALD R. BROWN
Bryn Mawr College

AN INTRODUCTION TO JUNCTION TRANSISTOR THEORY, by R. D. Middlebrook. 296 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1957. Price, $8.50.

R. D. Middlebrook, who teaches a graduate course on transistor electronics and directs a transistor research program at the California Institute of Technology, has demonstrated outstanding ability in preparing this new volume on transistor physics.

The book contains a resumé of the theory from earlier papers expressed in ways which make it directly applicable to circuit development and particularly to the design of semiconductor devices. Conduction in semiconductors is explained in terms of the structure of the materials, and the practical methods by which this structure can be altered are indicated. Conduction and mobility of charges are then related to gross,

measurable effects. These relations lead naturally to certain equivalent circuits which are of value to the engineer in visualizing the practical use of the device.

A modern curriculum for students of communications, physics, or industrial electronics could benefit from a course built around Professor Middlebrook's book. However, such a course would be given primarily to advanced students who have some acquaintance with classical and quantum mechanical theories. These topics find importance since the energy levels of the carriers in the transistor determine the character of the current flow.

In considerable degree the treatment by this author is mathematical, and hence some proficiency in that regard is required of the student. To those interested strictly in circuit designs the book might leave something to be desired, because there are no chapters on amplifiers, counting circuits, or voltage regulators as such. On the other hand, this was not the author's intent; the physics of internal capacitance, high frequency phenomena, and feed-back effects is, as the title implies, covered thoroughly. The connection between the analysis of these effects and the circuits mentioned above is easily made by the engineer.

One is immediately impressed by Professor Middlebrook's teaching skill. Of all the transistor papers and books that have come to this reviewer's attention, no other treatment is as understandable and yet penetrating. It may serve durably as both a text and a reference.

C. W. HARGENS

The Franklin Institute Laboratories

SOIL FERTILITY AND FERTILIZERS, by Samuel L. Tisdale and Werner L. Nelson. 430 pages, illustrations, 6 × 9 in. New York, The Macmillan Co., 1956. Price, $7.75.

The fundamentals of plant nutrition, crop fertilization, and soil management are presented in this text designed for the student in soil fertility. Each chapter is outlined logically and in detail, and each subject is treated in a clear and concise manner without resort to a confusing mass of data. The book reflects the experience of the authors in the fields of soil fertility research, soil testing, and teaching.

The text is organized in four parts. The first part provides a brief historical review of the development of soil fertility from the early Greek and Roman times to the present. The second part, consisting of six chapters, begins with a discussion of the genetic and environmental factors affecting plant growth. The mineral nutrient elements receive the greatest attention, each element being considered with respect to its role in plant nutrition and with respect to the forms and behavior of the element in the soil. The third part of the book is devoted to the source, manufacture, and properties of fertilizers, and their agronomic value. The old established materials are discussed, as well as many of the new high-analysis materials. The composition and use of farm manure and the problems of soil acidity and liming are also covered in this section. The last part is concerned with soil fertility evaluation by various means, the fundamentals of fertilizer application, the effects of cropping systems on soil management, and the economics of the use of lime and fertilizer. The final chapter, in which various factors are considered in attacking soil fertility problems, emphasizes the need for careful planning of any experimental work.

The questions at the end of each chapter are designed to bring out the important points in the text, and to relate the material to the student's own state or situation. The references cited are well chosen, and represent the latest findings in the various subjects at the time of writing. This book should be of great help to students and teachers of soil fertility.

L. F. MARRIOTT

The Pennsylvania State University

SCIENTIFIC USES OF EARTH SATELLITES, edited by James A. Van Allen. 316 pages, diagrams, 6¾ × 10 in. Ann Arbor, University of Michigan Press, 1956. Price, $10.00.

The stepping stone into space is the atmosphere. Before the scientist can intelligently plan the procedures for getting off the earth and into space it is necessary to gain complete knowledge of the atmosphere. While the ultimate goal is space travel, at the moment this goal is masked by an intense research on guided missiles which in turn necessitates thorough investigation of the atmosphere.

It is only for strictly utilitarian purposes such as guided missiles with their high speeds and reentry problems that monies can be spent for research.

Concentrated efforts by various groups have begun to deal with particular facets of the space travel picture. One such group is the Upper Atmosphere Rocket Panel constituted in 1946. Though an informal and unofficial body, it has been the coordinating agency for all atmospheric investigations.

Scientific Uses of Earth Satellites is a compilation of the 33 papers given at the tenth anniversary meeting of the Panel held at the University of Michigan. The meeting was devoted to the uses of small satellites to be launched within the next few months.

The roster of contributors to the volume is a most impressive one for many of these highly articulate men have all been contributing to the field since the advent of the rocket as a tool for upper atmosphere research. The range of topics covers every conceivable question raised concerning the satellites.

Dr. Fred L. Whipple leads off as the ranking member of a group which has studied the physical behavior of the satellite. The type of orbit and the duration of satellite visibility for a ground observer are covered.

Several contributors take up the problem of tracking the satellite. A particularly interesting feature of Hudson's paper is the seeing of the satellite by a self-contained high intensity blinking light. It has considerable merit and could be a short term solution to the problem of visibility.

Instrumentation, as is expected, plays a significant role in the satellite program and many of the papers contain sections which deal with specific instrumentation for special research. Jones and Bartman describe an accelerometer for drag studies; Stakutis and Brennan speak of a photronic cell for measuring sky brightness; Van Allen discusses equipment for measuring cosmic rays and auroral particles; Singer describes a nuclear precession magnetometer for measuring the earth's magnetic field, and Daniels describes the instrumentation for studying electromagnetic propagation. This is by no means a complete catalogue of the instruments designed for satellite experiments, these are merely an index to the type of devices which will be used.

In this comprehensive volume are found excellent treatments of theoretical aspects of the satellite problems. Gast presents an acute analysis of the temperatures which might be expected in the satellite. His Table IV summarizes the information he derives and from his studies, temperatures do not appear to be a problem. Air density measurements are covered by Jones and Bartman using accelerometer techniques and the determination of density by the characteristics of the satellite orbit and satellite orientation are detailed by Spitzer. Geomagnetic studies are outlined by Heppner, Katz, Singer, and Vestine.

Serious satellite studies are only a few years old and it is a tribute to the staging agency and to the editor that so much detailed information on satellites could be packed into so modest a book. It is the last word on studies which are to be made during the next two years.

In the preface the editor says, "The authors, the editor, and the publishers are well aware of the evanescent nature of descriptions of proposed experiments. Yet it has been felt worthwhile to make available in collected form the best early thinking in this subject. Much of the analytical work herein is of durable value."

In this the reviewer heartily concurs. The basic work contained in this volume will form a secure foundation for future research.

Scientific Uses of Earth Satellites is superbly written, brilliantly conceived and is unqualifiedly the finest book that has appeared on this subject.

I. M. LEVITT
The Fels Planetarium

FIBRES PLASTICS AND RUBBERS, by W. J. Roff. 400 pages, 5½ × 8½ in. New York, Academic Press, Inc., 1957. Price, $10.00.

Fibres Plastics and Rubbers is no ordinary handbook since it gives the reader a clearer appreciation of the common polymeric materials. By showing the relationship (with the aid of tables) of the physical properties of the polymerics to other related materials, the author conveys the impression that they are to be considered as fundamental industrial materials and not as substitutes.

The basic value of the book, however, lies in the extensiveness of general information

that it contains. The author discusses briefly the chemical structure, preparation, and identification (chemical and physical) tests of the various polymerics. Extensive data are also available on the general physical properties of the polymers. In conjunction with this, the author has discussed the various tests used for determining physical properties so that the reader has a better understanding of what they mean. For added interest, he has also included a brief history of each polymer and other miscellaneous information. MARVIN PACKER

The Franklin Institute Laboratories

SOLID STATE PHYSICS, VOLUME 3, ADVANCES IN RESEARCH AND APPLICATIONS, edited by Frederick Seitz and David Turnbull. 588 pages, diagrams, 6 × 9 in. New York, Academic Press, Inc., 1956. Price, $12.00.

It is not at all surprising to find that the third volume of this series continues to maintain the standards of excellence set by the preceding two. The present volume contains both surveys of some of the broader aspects of the field as well as detailed discussions of specific areas which are currently being vigorously investigated by solid state physicists.

H. Welker and H. Weiss discuss the structure and properties of "Group III-Group V Compounds." In addition to an analysis of the semiconduction properties of these compounds, the available thermoelectric and infrared absorption data are presented in some detail. The influence of the type of binding on the band structure is also examined.

The potentialities and limitations of "The Continuum Theory of Lattice Defects" are presented by J. D. Eshelby. The formal theory consists of describing the elastic energies associated with various types of defects in solids which are assumed to be essentially continuous media. Application of the theory is then made to point defects, dislocations, and surface and volume defects. The author demonstrates very adequately that ". . . the theory perhaps suffers from the disadvantage that its limitations are more immediately obvious than those of other approximate methods which have to be used in dealing with the solid state, for it sometimes gives good results even in what appear to be extreme cases."

The chapter on "Order-disorder Phenomena in Metals" by L. Guttman is a distinguished review of this topic. The various theoretical descriptions of the ordering process, as well as the principal experimental techniques are concisely and critically evaluated. Especially useful are sections on modifications of the theory of the Ising model, and the kinetics of order-disorder transformations. A list of experimental studies of the equilibrium properties of ordering systems is included in an appendix.

The mechanism of "Phase Changes" is reviewed by D. Turnbull. The topics considered are the classical theory of phase stability, the theory of the nucleation and growth of one phase in another, and the kinetics of phase transformations. The section interpreting the data on the kinetics of phase transformations is particularly stimulating.

"Relations between the Concentrations of Imperfections in Crystalline Solids" are presented by F. A. Kroger and H. J. Vink. This paper deals with the relations between concentrations of electrons and holes, vacancies, interstitials, misplaced lattice atoms, and foreign atoms in binary non-metallic compounds. The interaction between imperfections involving both atomic and electronic disorder are described in terms of the law of mass action. Various degrees of deviation from stoichiometry are considered in terms of the formal theory developed by the authors. This chapter is a very well organized and complete description of an important phase of the theory of compound crystals.

The chapter on "Ferromagnetic Domain Theory" by C. Kittel and J. K. Galt is for the most part, an adaptation of Prof. Kittel's review article in the *Reviews of Modern Physics* (1949). The excellent organization and manner of exposition of the original are preserved in the present paper. The principal additional material consists of discussions of the motion of domain walls and of some of the newer approaches to the problems of the influence of thermal fluctuations on domains and the mechanisms of domain nucleation. While there can be no argument with the authors' contention that the basic approach of domain theory has been justified experimentally, it might have been desirable to point out that the interpretation of most of

the data in this field has been successful only in a very qualitative sense. In particular, the role of point defects and gross structural irregularities in determining the coercive force of bulk materials and fine particles might have been emphasized to a greater degree. However, it is likely that an extensive discussion of these and other points would have resulted in an article of unmanageable length.

AMI E. BERKOWITZ
The Franklin Institute Laboratories

ELECTROCHEMISTRY, PRINCIPLES AND APPLICATIONS, by Edmund C. Potter. 418 pages, diagrams, 6 × 9¼ in. New York, The Macmillan Co., 1957. Price, $10.00.

Dr. Potter's aim in writing this book was to produce a "concise and introductory text. . . ." Students and technologists will probably find a strong appeal in the book's fusion of fundamental and technical electrochemistry. There is a very brief bibliography of collateral reading. This reviewer believes that a much more comprehensive bibliographical survey is in order for a book of this type. An introductory text should provide the reader with a means of access to at least a portion of some of the intensely interesting original works plus a well chosen sampling of current literature.

Throughout the text, the reader is assumed to have no more than an intermediate knowledge of chemistry and mathematics. Simple mathematical illustrations point up the text, which itself, many readers may consider oversimplified.

A brief treatment of ionization is given as an introduction to Faraday's Laws. Several electrode reactions and types of cell design illustrate the principles of coulometry. Electrolytic conductance is gone into at some length covering ionic conductance, ionic mobility, transport numbers, ionic hydration and complex ions. The shortcomings of the Arrhenius Theory are reviewed in light of modern concepts of ions in solution, particularly the Debye-Hückel-Onsager treatment of interionic attraction.

Dr. Potter's discussion of ionic activity is one of many in the text which is so clear cut as to defy misconstruing. It is such treatments of normally difficult passages of electrochemistry that will make this book valuable to students and to technologists whose background in this field is sketchy.

Chapter III is a fairly straightforward treatment of such material as acid-base equilibria, ionization of water, definitions of acid and base (Brönsted), pH and temperature, acid-base neutralization, hydrolysis of salts, buffer action and titrations and indicators.

Under Reversible Electrode Potentials are included a thermodynamic analysis of electrode potentials and some discussion of the Electromotive Series, oxidation-reduction reactions and liquid junction potentials.

An entire chapter is devoted to the requirements for a reference electrode and the principles and construction of several such systems are covered in detail. The following chapter, "Irreversible Electrode Processes," discusses the causes and effects of irreversibility which is characteristic of nearly every practical system.

Chapter VII, The Electrical Double Layer and Electrokinetic Phenomena, briefly discusses the various proposed theories on the distribution of electric charges in the immediate vicinity of the interface between two conducting phases including those of Helmholtz, Gouy-Chapman, Stern and more recently Devanathan. This chapter further covers the measurement of double layer capacity and goes into electrokinetic phenomena or electrical layers which are not confined to the interface between electrically conducting phases including streaming potential, electro-osmosis, electrophoresis, and the Dorn Effect.

Two chapters are devoted to Electrochemical Measurements and Their Analytical Applications. An attempt is made to illustrate the application of fundamental electrochemistry to the analysis and solution of industrial problems by translating "some changing property of a reacting system into an electrical impulse. . . ." The material includes measurements of electrical conductivity, electrode potential, pH, potentiometric titration, polarography, amperometric analysis, electrogravimetry, coulometric analysis and ion exchange.

A theoretical treatment is given which relates corrosion processes and their electrochemical mechanisms. Corrosion inhibitors are touched upon briefly. A second chapter

deals with the practical aspects of corrosion and means of control. Phosphating, painting, metallic coating, cathodic protection, application of some inorganic inhibitors, and use of various metal alloys for specific applications are reviewed.

Considerable attention is given to electrolytic processes. Economics and efficiency are discussed. Under cathodic processes the author covers electroplating, electrowinning, electrorefining, electroforming, production of metal powders, and electrolysis from fused salts. The latter topic is covered in some detail including operation and design, decomposition potentials, production of aluminum and magnesium, electrolysis of molten caustic and molten sodium chloride. Finally, under cathodic processes, Dr. Potter includes production of hydrogen, electroreduction, preparation of caustic soda and cathodic cleaning.

Under anodic processes, the production of gases, metal purification, anodic oxidation, electropolishing, anodizing and electrocleaning are considered.

The last two chapters cover primary and secondary storage cells in the same clear cut and very basic analytical approach used throughout the text. WILBUR S. HALL

American Chemical Paint Co.

ANALYSIS OF BISTABLE MULTIVIBRATOR OPERATION, by P. A. Neeteson. 82 pages, diagrams, 6 × 9 in. Te Eindhoven (Holland), Philips' Gloeilampenfabrieken, 1956. Price not given.

The bistable multivibrator, originally conceived by Eccles Jordan in 1919, has with the advent of electronic computers and counters achieved a place of unique importance in the realm of electronics. The present text, written as a thesis for a doctor's degree, seeks to present a thorough analysis of the dynamic behavior of this circuit element. Previous analyses have dealt mainly with the stability of the static condition. Dr. Neeteson has endeavored to derive formulae for the phenomena occurring when the multivibrator is switching over, taking account of both tube characteristics and circuit components.

Three phases of the switching period are defined. The first starts at the instant the triggering potential is applied. Assuming the conducting tube to be instantaneously cut off, this is the period of time during which both tubes may be considered non-conducting. Phase 2 begins the instant the second tube becomes conducting, new transients being superimposed upon those of Phase 1. The final phase considers phenomena occurring at the instant the second tube commences to draw grid current. Complete expressions for the voltage changes at the terminals of the tubes have been derived.

In this little book, Dr. Neeteson has included a brief account of the history of the subject as well as a survey of pertinent literature with synopses of important papers. A number of appendices provide detailed explanations of the critical points of analyses outlined in the main text. Theoretical findings are well backed up with experimental results.

The text, evidencing a great deal of original work on the part of the author, provides a complete and valuable treatise on bistable multivibrators. It is only through such thorough analyses that one can expect to achieve the optimum in the design of equipment and circuits utilizing these devices. In the words of the author, "This book may be appreciated by those who feel that in the long run it is not satisfactory to apply certain circuits on the basis of technical feeling or practical experience, but that a thorough insight into the nature of the phenomena is necessary to realize the full potentialities of these circuits."

D. L. BIRX

The Franklin Institute Laboratories

CELLULAR MECHANISMS IN DIFFERENTIATION AND GROWTH, edited by Dorothea Rudnick. 236 pages, illustrations, 6 × 9¼ in. Princeton, Princeton University Press, 1956. Price, $7.50.

This volume is based on 11 papers presented by eminent biologists at the 14th Symposium of the Society for the Study of Development and Growth. The investigations reported are approached from a wide variety of theoretical and technical standpoints. Among the fields of research which are covered are botany, embryology, genetics, immunology, microbiology, and neurology. The biological

specimens studied are from many phyla of both the plant and animal kingdoms. The principal contributors include M. Delbrück, Hewson Swift, I. Manton, Vance Tartar, Joshua Lederberg, Maurice Sussman, M. C. Niu, Ralph H. Wetmore, Viktor Hamburger, Betty Ben Geren, and R. E. Billingham.

Four of the chapters are on intracellular events and structures. These discussions are on events occurring in single cells, some of which were investigated by means of spectrophotometric and electron microscopy and microsurgery. The titles of the chapters are: I. System Analysis for the Light Growth Reactions of Phycomyces; II. The Cytology of Nuclear RNA; III. Plant Cilia and Associated Organelles; IV. Pattern and Substance in Stentor.

The other seven chapters deal with interrelations among differing cells. Consideration is given to the long recognized phenomenon that two different types of cells may affect each other by contact or more remotely by production of diffusible substances. The specific subjects discussed are: V. Infection and Heredity; VI. Cellular Interactions during the Development of the Cellular Slime Molds; VII. New Approaches to the Problem of Embryonic Induction; VIII. Growth and Development in the Shoot System of Plants; IX. Developmental Correlations in Neurogenesis; X. Structural Studies of the Formation of the Myelin Sheath in Peripheral Nerve Fibers; XI. Acquired Tolerance of Foreign Cells.

This book should be of value to advanced students of biology and to research workers interested in problems of growth and differentiation.

W. D. YUSHOK

The Biochemical Research Foundation

CHEMICAL ENGINEERING PRACTICE. VOLUME II: SOLID STATE, edited by Herbert W. Cremer and Trefor Davies. 655 pages, diagrams, 6 × 9¼ in. New York, Academic Press, Inc.; London, Butterworths Scientific Publications; 1956. Price: $17.50 (non-subscription); $13.50 (subscription, valid on orders for complete set of 12).

The authors have wasted no time with a Preface (which probably few readers appreciate) to this second volume of a series of twelve in which they propose to cover the gamut of materials, practices, and scientific disciplines with which the chemical engineer is confronted. They began this effort with a refreshing summary of "Fundamental Concepts of Matter." While the treatment of the various aspects of structure and properties of matter must be arbitrary in covering such a comprehensive subject, it was felt that more information might have been given on semiconductors in view of recent developments.

Chapter 2 on the "Significance of Mechanical Properties and their Measurement" serves as a good review of the techniques used and their value in determining the various properties. The chapter is concluded with an excellent select bibliography.

"Alloy Equilibrium Diagrams" are described briefly in Chapter 3 and supported by an adequate reference list. "The Range of Steels" in Chapter 4 could be much greater than the author indicated but he may have been limited by short supply of space. "Fatigue in Metals" has been an intriguing and baffling problem and the chemical engineer should find this treatment stimulating.

"Creep in Metals" is probably one property of materials which has had more attention focused on it in recent years from more exacting design requirements. This chapter reflects the increased interest including an impressive set of references. The importance of plastic materials and glasses warrant far greater stature than allotted in Chapter 7. No mention is made of fiber glass and its combination with various resinous products. The chemical industry in particular, is finding some of the polymeric materials well suited for certain corrosive resistant applications in pipes as well as coatings.

Undoubtedly, one of the most important responsibilities of the chemical engineer embraces that of selection and maintenance of systems to withstand the onslaught of "Corrosion of Metals" with its contamination. The presentation of the various types of corrosion and suggested mechanisms is very comprehensive. The 291 references would be a long winter's study of the subject.

The editors have chosen to give "Porous Masses" the most comprehensive treatment of any subject in this volume. This may be

justified on the basis of the many applications of the basic principles in the oil, gas, and other process industries where fluid flow through porous masses is a controlling factor. The chapter then becomes a necessary study for the subsequent considerations of "Porous Powder Metallurgical Products for Filtration and Related Uses" and "Powder Metallurgy." The utility of these products and processes is unique for certain solid and fluid separations and applications where fabrication costs can be greatly reduced.

The "Purification of Coal Gas," "Flow in Fuel Beds," "Use of Porous Masses in the Treatment of Water and Sewage," and "Transpiration Cooling" are specialized aspects of the earlier chapter on "Porous Masses."

Since over one half of the volume has been devoted to porous masses and fluid flow through them, the title might logically include porous masses. In general, the editors have assembled indispensable information in a fashion which a practicing chemical engineer can readily use. They have avoided any but the necessary duplication and this is well cross-indexed. Each author has contributed a good number of references with a select bibliography in special instances so that the reader can go deeper into the subject. The chemical engineer, in teaching as well as in practice, should find this volume a worthwhile addition to his reference library.

R. S. DALTER
The Franklin Institute Laboratories

QUALITY CONTROL FOR PLASTICS ENGINEERS, edited by Lawrence M. Debing. 139 pages, diagrams, 6 × 9 in. New York, Reinhold Publishing Corp., 1957. Price, $4.95.

The basic concept of SQC (Statistical Quality Control) is supplied in the introduction—"control a process by examination of the product, rather than to control the product alone." The various techniques available for solving product control problems have been simply explained, each by an expert in that particular phase of SQC and in such a fashion as to enable the reader to apply them for his use. Each author has shown with the aid of illustration where a particular technique is eminently suitable, and its advantages and shortcomings. References are supplied at the end of each chapter for those plastic engineers, research technical personnel, and production men who are interested in delving more deeply into a particular technique of SQC.

The reviewer was also impressed by the easy-to-read print used in this book.

MARVIN PACKER
The Franklin Institute Laboratories

INTRODUCING ASTRONOMY, by J. B. Sidgwick. 259 pages, diagrams, 5¼ × 8 in. New York, The Macmillan Co., 1957. Price, $3.50.

With all the high-powered books that have appeared on the astronomical horizon it is refreshing to find a book written for the uninitiated which is comprehensive and authoritative.

The book is superbly written by a man who looks on the entire sky as something to understand. It is divided into two sections. The first is a survey of the objects in the sky—the sun, moon, planets, stars, galaxies, etc. A unique and welcome addition is a brief description of the principal features on the moon.

The second section is devoted to the constellations. A diagram of the constellation showing its orientation with the principle stars named is given. The mythology and history of the constellation are given as are the objects of interest in the constellation. This section is much too brief but still ample for the beginner.

Introducing Astronomy most definitely deserves its place on the book shelves.

I. M. LEVITT
The Fels Planetarium

ARCS IN INERT ATMOSPHERES AND VACUUM, edited by W. E. Kuhn. 188 pages, illustrations, 8½ × 11 in. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1956. Price, $7.50.

This book is a collection of papers presented at the Symposium on Arcs in Inert Atmospheres and Vacuum sponsored by The Electrochemical Society. After an introduction by the editor covering the past, present and future of controlled atmosphere arcs, there are thirteen papers presented in

this edition. These papers are divided into three sections: Arc Melting Fundamentals; Design and Operation of Arc Melting Furnaces; and Chemical Applications of Arcs.

With the ever-growing demand for the higher melting metals and high purity metals and alloys, the field of arc melting is of prime importance to the metallurgical industry today. It is for this reason that great advances have been made in the technology of arc melting within the last few years. This book is a very fine presentation of the current status of the knowledge and techniques in this field.

In the section on Arc Melting Fundamentals, the properties of arcs in the rare gases and in vacuum are dealt with. There is a very interesting discussion presented on the behavior of the arc as a function of pressure, elucidating the problem of arc stabilization at reduced pressures. There is included a paper presenting the effects of variables, such as kind of metal, crucible to electrode size ratio, power input, etc., on the melting rate in the consumable electrode arc furnace. The final paper in this section deals with the effects of magnetic stirring on ingot quality.

In the third section, several designs for different types of arc melting furnaces are presented, including discussions on the various types of electrode control systems. A paper on German developments in this field is included, which contains an interesting discussion of the hazards involved in the operation of arc furnaces.

The final section is concerned with a very important application of arcs for the future. This is the utilization of arcs for the reduction of ores of metals which are difficult or impossible to reduce by other means, for example, the rare earth metals. The reduction problem with its inherent difficulties is discussed along with several other applications of the arc to process chemistry.

As the editor mentions in his introduction, a good portion of the information presented in this field is obsolete almost as soon as the material is published because of the rapid advancement of this technology. Nevertheless, this book is a valuable contribution to anyone interested in the present status of the techniques of arc melting.

G. E. SPANGLER
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PUBLICATIONS RECEIVED

The Editors wish to call attention to these new books, received in the JOURNAL office for review. Lack of space prevents publication of more than this listing.

VIBRATION ANALYSIS TABLES, by R. E. D. Bishop & D. C. Johnson. 59 pages, 8½ × 11 in. New York, Cambridge University Press, 1957. Price, $2.00 (paper).

APPLIED MATHEMATICS IN CHEMICAL ENGINEERING, by Harold S. Mickley, Thomas K. Sherwood and Charles E. Reed. Second edition, 413 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1957. Price, $9.00.

THEORY OF VIBRATIONS FOR ENGINEERS, by E. B. Cole. Third revised edition, 362 pages, diagrams, 5½ × 8½ in. New York, The Macmillan Company, 1957. Price, $6.00.

HEAT AND THERMODYNAMICS, by Mark W. Zemansky. Fourth edition, 484 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1957. Price, $7.50.

URANIUM PROSPECTING, by Hubert Lloyd Barnes. 117 pages, 5½ × 8 in. New York, Dover Publications, 1957. Price, $1.00 (paper).

THEORY OF LIE GROUPS: 1, by Claude Chevalley. Reissue of out-of-print volume in the Princeton Mathematical Series. 217 pages, 6 × 9 in. Princeton, Princeton University Press, 1957. Price, $2.75 (paper).

INTEGRATION, by Edward J. McShane. Reissue of out-of-print volume in the Princeton Mathematical Series. 394 pages, 6 × 9 in. Princeton, Princeton University Press, 1957. Price, $2.95 (paper).

BOOK NOTES

THE ENCYCLOPEDIA OF CHEMISTRY, edited by George L. Clark and Gessner G. Hawley. 1037 pages, 6¾ × 10 in. New York, Reinhold Publishing Corp., 1957. Price, $19.50.

Truly a remarkable achievement is this encyclopedia containing contributions from over 500 authorities in the field of chemistry and 20 allied sciences. Included are elements, compounds, products, scientists, processes, agencies, institutions, etc.—each treated accurately and concisely. Most entries are at least half a page in length, and all are signed by the contributor. One of the best features is a system of cross-referencing, whereby the reader is guided to other related subjects treated in the volume. As an example of the various subjects covered, these are picked at random: cutting fluids, esters, Robert Hooke, paper, saponification, Tungsten Institute. The index distinguishes between "main" articles and text references by means of boldface and roman type, respectively.

Well bound, clearly printed with legible type on good quality paper, this volume is a real credit to its editors and to the publisher. It will be a valuable addition to the general library of any manufacturing concern, as well as to those of strictly chemical interest.

WEATHER MAP. Fourth edition, 105 pages, illustrations, 6 × 9¼ in. London, Her Majesty's Stationery Office, 1956. Price, 10s. 6d.

First published in 1916, *Weather Map* has been a "best seller" through three editions. This new fourth edition, incorporating the latest forecasting techniques, should prove just as popular as its predecessors. Large portions of the book have been rewritten; a new set of weather maps is included; and modern forecasting techniques are discussed, using maps from which actual forecasts were made. The book is aimed at the intelligent layman, interested in weather forecasting.

THE LEIBNIZ-CLARKE CORRESPONDENCE, edited by H. G. Alexander. 200 pages, 5 × 7¼ in. New York, Philosophical Library, 1956. Price, $4.75.

The full correspondence between Clarke and Leibniz concerning Newtonian "philosophy" has not been published in English since 1738, until this present edition, edited by H. G. Alexander. In addition to the letters, passages from the *Principia* and *Opticks* are included when referred to; also, references to the controversy have been collected from Leibniz' other correspondence and are appended to the main portion of the book.

This scholarly edition, with notes and an introduction by the editor, will be welcomed by historians of science.

MECHANICAL ENGINEERING LABORATORY INSTRUMENTATION AND ITS APPLICATION, by Jesse Seymour Doolittle. 393 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., 1957. Price, $6.50.

This new text for a first course in mechanical engineering laboratory is the first to separate laboratory work on instrumentation from that for engineering tests. The text stresses the uses and limitations of the various types of instruments used in mechanical engineering. The book is divided into three parts; Instrumentation; Determination of Selected Physical and Chemical Properties of Fuels and Lubricants; and Engineering Experimentation. Each chapter concludes with suggested exercises, questions and problems.

NOTES ON ATOMIC ENERGY FOR MEDICAL OFFICERS. 169 pages, plates, 4¼ × 7 in. New York, Philosophical Library, 1956. Price, $4.75.

The staff of the British Royal Naval Medical School prepared this monograph of the effects—both long-term and short-term—of atomic radiation. Intended for use by

medical officers, over half the text covers introductory chapters on physics, radioactivity, ionization, and the atomic bomb. The medical chapters review the life history of the cell, the effects of radiation on cells and on the body as a whole, and one chapter (five pages) discusses treatment of radiation casualties. A short chapter on monitoring instruments and one on protection against radiation conclude the main portion of the book. Several appendices give useful information on allied subjects (for example, "Uses of Isotopes" and "Maximum Permissible Hazards for Industrial Purposes").

ATOMIC ENERGY FOR YOUR BUSINESS, by Arnold Kramish and Eugene M. Zuckert. 269 pages, 5½ × 8 in. New York, David McKay Co., Inc., 1956. Price, $3.95.

The authors wrote this book for the business man who is faced with the problem of deciding whether atomic energy can be used to advantage in his own plant. After three introductory chapters—in which atomic energy is explained in elementary (but accurate) terms—the reader finds a lucid description of the process of releasing atomic energy through a nuclear reactor. The specific uses of atomic energy in industry are set forth in a chapter on "Applications of Atomic Energy," and the following chapter gives specific cases of "The Atom at Work in Industry." The text is supplemented by five appendices occupying nearly one third of the book, giving more technical information on reactors, fuels and hazards than is found in the main body of the book.

The book is recommended as good background material for the business man who is wondering whether there is a place in his company for atomic energy.

METAL CASTING, by Joe L. Morris. 296 pages, illustrations, 5½ × 8½ in. New York, Prentice-Hall, Inc., 1957. Price, $7.35.

A staff member at Los Alamos Scientific Laboratory, the author has prepared this book to "describe and discuss all current and potentially valuable metal casting procedures." In order to cover the ground, he has emphasized principles and omitted discussions of procedural detail. The first chapter serves as a preview and an introduction, for it briefly describes many of the processes to be developed more fully in later chapters. The book includes current practices and is illustrated with numerous schematic drawings and photographs. Written in clear, concise language, the book should be of interest to practically all industries—for casting is an important process in a great many different fields.

MACHINE TOOLS FOR METAL CUTTING, by W. H. Armstrong. 347 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1957. Price, $5.00.

A practical text, this new book is designed for a one-semester course in machine tools for metal cutting. Introductory chapters define and classify machine tools, describe the measuring tools used in machining, define "metal fits," and describe the metal-cutting process. The main portion of the book deals with general-purpose metal-cutting machine tools for all the processes (turning, milling, drilling, grinding, planing, etc.). Specialized subjects such as ultrasonic machining, chem-milling, automation, screw-thread production and gear production are treated in individual chapters. Well illustrated and printed in large type (leaded for easier reading), the book should be useful in machine shops as well as in the classroom.

APPLIED STRENGTH OF MATERIALS, by Alfred Jensen. 343 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1957. Price, $5.75.

Designed as a text for a one-semester course on strength of materials, this book gives a simplified approach to the subject, omitting the use of calculus in all design formulas. This elimination of calculus makes it possible to use the book in industrial training programs and in technical institutes. Each chapter is divided into articles, each presenting a new idea, a new application or a new theory. Each of the articles is accompanied by at least one illustrative example. At the end of each chapter there is a section of review problems, with about a third of the answers supplied. The author will supply

answers to the rest of the problems at the request of the instructor.

ELEMENTS OF ENGINEERING THERMODYNAMICS, by Rolf H. Sabersky. 318 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1957. Price, $7.50.

The author, who teaches at California Institute of Technology, designed this text for a first course in thermodynamics and included just enough material so that major omissions are not necessary. Basic laws are covered and their experimental foundations discussed. Examples of applications to engineering are drawn from those industries employing the greatest number of engineers. The usual subjects are treated—First and Second Laws, Properties of a Perfect Gas, Cycles (Otto, Diesel, gas-turbine, steam-power, jet-propulsion), flow, turbine design, combustion. For those wishing to supplement the text, references for further study are given in Appendix B, listed by topics.

MECHANISMS AND DYNAMICS OF MACHINERY, by Hamilton H. Mabie and Fred W. Ocvirk. 442 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1957. Price, $8.50.

Used for several years in mimeographed form at Cornell University, this text is suitable for two consecutive three-hour courses. Some topics which are not usually included in undergraduate texts are treated, such as analytical cam design, non-standard spur gears, computing mechanisms, synthesis and vibrations. In order to include these topics, presentation of the standard topics has been shortened. Since prerequisites for the course include physics, calculus, statics and dynamics, derivations of fundamental relationships are not repeated in this text.

Transistorized Computer.—The Philco TRANSAC, Type S-1000, is a large scale electronic computer compactly designed for scientific application in research laboratories, various industries, and universities. Its new speeds and extensive command structure provide for the solution of lengthy and complex mathematical problems. Philco's pioneering leadership in airborne control systems has been carried over into the design of the wholly transistorized, printed wiring TRANSAC S-1000.

The highly developed and extremely reliable Philco transistors, unlike vacuum tubes, require no periodic replacement. The transistor circuitry eliminates bulky insulation and the heavy power supply equipment associated with vacuum tube devices.

Generating comparatively little heat, the S-1000 requires only a small fraction of the air conditioning equipment normally required for large scale computers. In fact, only the amount of cooling normally needed for human comfort is sufficient.

The TRANSAC computer entails no installation work or special wiring. It may be plugged into existing 110 volt, 60 cycle outlets and is equipped with castors, making it highly mobile. Thus extreme compactness, light weight, low power consumption, high speed and reliability are now realized in this desk model computer. These unique features, plus the proven system philosophy and command structure, provide a new and powerful tool for science and research.

FEATURES OF S-1000

Machine Proper

1. Parallel Machine.
2. 36-Binary Digit Word Length, Double Length 72 Binary Digit Accumulator.
3. One's Complement Binary Arithmetic for Internal Calculations.
4. Two-Address Instructions with Two 12-Digit Addresses, Two 3-Digit Address Modifiers, and a 6-Digit Command.
5. Magnetic Core Memory with 20 Microsecond Cycle Time.
6. Memory Capacity of 4,096 Words.
7. Arithmetic Speeds (exclusive of Memory):

   Addition or Subtraction: 5.5 μs.

   Multiplication:
   Average. . . . 130 μs.
   Maximum . . 200 μs.

   Division: 200 μs.
8. All Transistor Circuits in Arithmetic and Memory Sections.
9. Volume of Arithmetic Section, Memory Section, Power Supplies, Control Panel and Ventilating Equipment occupies about 36 cubic feet.
10. About 1.2 kilowatts in power.
11. Cathode Ray Memory Address Reference Indicator.

Auxiliary Devices

1. High Speed Perforated Tape Reader and Punch at 60 Characters Per Second for either 5 or 8 Level Tapes.
2. Magnetic Tape can be added, if desired.
3. Teleprinter is provided for manual insertion and read-out.

Wind Tunnel for Study of Surface Roughness.—Results of basic research by a University of Maryland scientist are expected to enable designers to predict the speeds of airplanes and ships before the designs leave the drafting board. Dr. Francis R. Hama, an assistant research professor in the university's institute for fluid dynamics and applied mathematics, has completed a new wind tunnel for the study of surface roughness on the sides of airplanes or ships as it affects friction resistance of water or air.

According to Dr. Hama, the sides of airplanes or ships are both smooth and rough. Rough areas are caused by paint, welding, or other protrusions.

"In our wind tunnel, we will be able to compare the velocity of air or water near a smooth surface to velocity near a rough surface," Dr. Hama explained.

"Through this measurement, it will be possible to determine the effect of surface roughness on friction resistance. This will then provide a correct prediction of the speed of a moving object," he concluded.

The test section of the wind tunnel, designed by Dr. Hama, is approximately 50 ft. long, 5 ft. wide and 4 in. high. Air, generated by a 50-hp. motor, will be forced through the tunnel at a speed of more than 100 ft. per sec.

Surface roughness of the type found on the sides of airplanes and ships will be simulated on the sides of the laboratory wind tunnel when further experiments are begun.

Sponsored by the Office of Naval Research, the study is technically entitled, "the effect of the surface roughness on turbulent shear flow."

The surface roughness finding could also aid in weather predictions, oceanography, meteorology and agriculture.

Atomic Power Plant for First Guided Missile Submarine.—Westinghouse Electric Corporation under contract with the U. S. Navy is designing a nuclear propulsion plant, for an atomic-powered submarine capable of carrying and firing guided missiles.

This guided missile submarine will utilize a submarine fleet reactor type nuclear propulsion plant which Westinghouse is designing for fleet type submarines authorized in the Navy's Fiscal Year 1955 and Fiscal Year 1956 Shipbuilding Program.

Westinghouse has already supplied the nuclear power plant for the world's first atomic submarine, the *U. S. S. Nautilus*, and is now working on the first nuclear power plant for a large surface vessel.

Valves for Reactors.—The American valve industry, which has cooly solved the problems of piping such thermally hot fluids as saturated steam and cracked petroleum, is warmly engaged in solving the problems of piping such radioactively "hot" fluids as molten sodium and dissolved plutonium.

Delegates to the 1957 Nuclear Congress in Philadelphia heard how a variety of ingenious fabrication and inspection methods were making it hot for the radioactive gremlins who want atomic power-plant valves to corrode, have seal leaks, and be porous.

It is vital to catch and remedy all such defects before valves are installed in the piping systems of atomic-energy plants, lest the spread or loss of radioactive fluids cause reactor "scram"—the sudden and expensive shutting-down of a nuclear reactor for safety reasons.

Almost literally fighting fire with fire, valve engineers use deadly gamma rays emitted by radioactive cobalt-60

to make radiographs that, like those made with X-rays, reveal internal valve defects invisible to external inspection.

Going to the other temperature extreme, valve engineers resort to freezing cold in their attempts to keep atomic power-plant valves from developing leaky seals. Some of the radioactive fluid to be piped—as, for instance, liquid sodium—may be frozen to form the valve seal. This seems like a practical application of the old physics-classroom riddle, "What would you keep the perfect solvent in?" to which the answer is, "In a container frozen from itself."

Such extreme recourses have necessarily meant that atom-power valves and pipe fittings cost more than their less critical counter-parts in normal-power piping. In a report read to the 1957 Nuclear Congress, the engineer J. J. Kanter said that much of the expense being built into these valves is dictated by caution.

Mr. Kanter's report was sponsored by The American Society of Mechanical Engineers, one of the 25 engineering and scientific groups participating in the congress.

"As experience is gained," Mr. Kanter went on to say, "it is to be expected that piping components utilizing . . . designs lending themselves to economical production will emerge."

Electronic Color Photography Predicted.—A high-speed system of electronic color photography that would provide permanent prints of pictures taken less than five seconds earlier at points thousands of miles away has been predicted by Dr. Irving Wolff. Vice-President, Research, of the Radio Corporation of America.

Speaking to the Boston section of the American Institute of Electrical Engineers, Dr. Wolff pointed out that techniques now exist for performing all steps in the photographic process by instantaneous electronic means "from exposure to final print, and including the means for storing images in electrical form on magnetic tape."

"We can now foresee future systems in which television cameras, magnetic tape recording equipment and electronic printing techniques will be linked in various ways to provide versatile high-speed chains with many applications in the home, in industry, and in defense," said Dr. Wolff.

"With such a system, for example, a newsworthy event in New Orleans, San Francisco, Chicago or another city might be viewed by a portable television unit carried by a single newsman. The image would be transmitted instantaneously from the portable unit to a nearby relay point for transmission over a closed circuit or network facilities to New York or Boston, where the image could be recorded electronically on magnetic tape as it arrived.

"The tape recording could provide a motion picture record for instant viewing or later playback and rebroadcast. At the same time, any single section of the tape, corresponding to a single frame of the television picture, could be played back repeatedly at high speed to provide a 'still' that could be printed electronically on paper directly from a special type of television picture tube. In this case, the entire process, from exposure in the distant city to the final print in New York or Boston, would take some seconds."

Dr. Wolff told the engineering group that the major elements of such a system already exist in basic form. He listed these as:

A compact, light-weight television camera, employing a new ½-in. Vidicon pickup tube and transistorized circuits. While present developmental types are designed for black-and-white television, he stated that portable color cameras incorporating these small components "are certain to appear in the near future."

The magnetic tape system for color television, which makes possible the recording of color or black-and-white signals for immediate or later playback.

The RCA "Electrofax" process of high-speed, direct printing by electronic means. Dr. Wolff pointed out that progress will be made in adding color to the "Electrofax" process to achieve color prints on paper, and that development is proceeding on a high-speed system for printing directly onto "Electrofax" paper from a specially-designed television-type tube.

"These comprise all of the elements of a photographic system," said Dr. Wolff. "However, a system employing these electronic steps would be totally novel in its over-all ability to handle motion and color, and to transmit visual information instantly to remote points for high-speed recording."

Looking into the more distant future, Dr. Wolff predicted the ultimate development of extremely compact systems for transmitting and storing pictures.

"In any system using light directly, we are limited by the wavelength of visible light in our ability to reduce the area for storage," he said. "Electronic methods, free from this limitation, give us the potential ability to store far more information in the same area. In addition, electronic methods of high-speed coding will be able to eliminate redundancy in transmitting and storing visual information. Already, electronic coding techniques have been applied with great effect to cut down such redundancy in verbal information. Potentially, therefore, we have a further means for increasing substantially the amount of picture information that can be stored in a given area."

Radioactive Waste Discharged Safely Into Soil.—The taxpayer has been saved more than a half-million dollars by the discovery that some radioactive wastes can be dumped into the ground at the Atomic Energy Commission's Hanford plant. Soil conditions peculiar to the arid portion of southeastern Washington where the Hanford plutonium plant is located have made it possible to discharge millions of gallons of liquid waste into the ground.

A paper prepared by two General Electric scientists at Hanford for presentation to the 1957 Nuclear Congress in Philadelphia describes the waste disposal operations at that atomic plant. The authors of the paper are M. J. Stedwell, Chemical Processing Department, and R. E. Burns, Hanford Laboratories employee of General Electric, prime contractor of the Atomic Energy Commission for the operation of the vast Hanford plutonium plant.

Large volumes of radioactive wastes are generated at Hanford in the processes of separating and decontaminating uranium and the man-made element, plutonium. These wastes are disposed of by various means depending on their level of radioactivity. Some wastes, such as process vessel cooling water, may be discharged to open areas and permitted to seep into the ground while others must be stored expensively in large underground tanks.

The reprocessing of these stored wastes could be the beginning of cesium recovery. Cesium is one of the basic isotopes which are finding increasing use in medicine, industry and agriculture.

Before wastes of even a low level of radioactivity could be poured onto the desert floor the scientists had to learn a great deal about the climate and geology of the area. This portion of Washington is semi-arid with a rainfall average of about seven inches per year. Summers are hot and dry and most of the rainfall occurs in the winter months.

The site of Hanford was at one time a large inland lake which has been filled subsequently with alluvial sand and gravel to a depth of 250 to 300 feet above the water table. In some places, remnants of a fine silt layer deposited while the region was covered with water remain above the present water table and act as a barrier toward movement of surface water to ground water.

Although the mighty Columbia River is only a few miles away, tests show that the movement of ground water toward it is slow. Travel time from the point where wastes are discharged to the Columbia have been estimated to range from 50 to 1,500 years, depending on the direction of flow.

The sand and gravel through which the wastes must pass to reach ground water serves as an ion exchange column to remove much of the radioactive material. From there, it takes many years to reach the Columbia River where the wastes may come in contact with plants or animals.

Under these conditions, radioactive elements of short half-life are of little concern. Among the long-lived isotopes, only cesium and strontium exist in solution. If radioactive wastes are discharged in such manner that neither of these reach ground water in hazardous amounts, the system is safe.

It is an established objective that radioactive wastes shall be discharged in such manner that the ground water will not be contaminated in any place. The GE scientists pointed out that the process has limitations as to the sites where it may be used and to the waste solutions for which it is applicable.

Because the wastes after treatment still contain relatively large amounts of radioactivity they would not be satisfactory for discharge to ground unless the combination exists of a long soil column capable of holding cesium and strontium and a long retention time in the ground water prior to any possible use of the ground water by plants or animals.

Subminiature Aluminum Foil Electrolytic Capacitors.—Designed expressly for transistorized and printed circuits, and other compact or miniaturized low-voltage d-c. equipment, the new Cornell-Dubilier "Electomite" Type NL electrolytics meet the demand for ultra-compactness in bypass, filter and coupling applications.

Available in d-c. working voltages of 1, 3, 6, 10, 15, 25 and 50 volts, in capacitances from 1.0 to 200 Mfd., and in sizes from $\frac{3}{16} \times \frac{1}{2}$ in. to $\frac{3}{8} \times 1$ in., these units can be operated within the temperature range of $-20°$ to $+85°$ C.

Advanced techniques in manufacture provide these units with extremely low d-c. leakage current and a far superior shelf life, assuring dependable performance and long operating life.

"Electomite" capacitors are compression-sealed, and resistant to moisture and humidity. Terminal leads are tinned copper wire. The

positive lead is connected to the anode through the rubber bushing of the compression seal, and the negative lead is grounded to the aluminum can. Also available with outer plastic insulating sleeve where physical contact with adjacent components is a design requirement.

Testing Tension of Tapes and Ribbons.—A hand-held meter which accurately measures tape tensions for use with magnetic tape recorders has been released by Tensitron, Inc., Harvard, Mass. The same instrument can also be used by manufacturers of typewriters, adding machines, and users or makers of narrow fabrics, ribbons, carton tape, and film.

The Tension Meter has a range of 0–1000 grams with great resolution in the range of 0–200 grams. It is used extensively in measuring tape transient tensions, such as encountered in sudden braking and starting, during coating, and other processing operations. The engineer who must design proper tensions into the tape recorder so that tapes are neither stretched nor broken in use will find it simplifies his job. The weaver, dyer, and user of narrow fabrics will also find that the checking of tension during manufacture avoids breaks and speeds up processing.

Measurements are made by pressing the trigger, inserting the tape, and then reading the tension upon release of the trigger. The face of the meter is easily read while the tape is running, stopping, or accelerating, since the dial faces the observer.

Contrary to belief, tape tensions as high as 900 grams have been found where no tensions were figured to exist. Proper tension affords maximum fidelity in tape recording and play-back. The Tensitron meter eliminates guess work as to tension.

Another important application is the use of the Tape Tension Meter for checking tapes and foils in the manufacture of electrical condensers. Since capacity follows the inverse square law with reference to the distance between the conductive surfaces and since the pressure with which the foil is wound is influenced by tension, greater accuracy in winding can be achieved by tension-sensitive indicating devices. Improvements in winding controls based on same can then be established thereafter.

Anticonvulsant Proven Effective in Psychomotor Seizures.—Parke, Davis & Company has announced a new anticonvulsant for treatment of patients with certain types of epilepsy. The drug, Celontin, has been found to be effective in treatment of psychomotor seizures, the company's announcement said. A decade in developing, Celontin also has proved effective in reducing frequency of seizures among patients whose petit mal attacks were not controlled by other medications.

Dr. Loren M. Long, director of chemical research, and Charles Miller were responsible for synthesizing Celontin, which is available in Kapseal form in bottles of 100 only by prescription.

The drug (N-methyl-*a*, *a*-methylphenylsuccinimide) was studied by Dr. Graham M. Chen, laboratory director in pharmacodynamic research, and Charles Ensor, research pharmacologist. They spent approximately five years screening hundreds of compounds before Celontin emerged as the most effective. Another five years was required to make sure Celontin could be tolerated by humans.

Celontin is the latest of Parke-Davis' anticonvulsants. The company now has one such product for each of the three major types of epileptic seizures: Dilantin, for Grand mal, psychomotor; Milontin, for Petit mal; and Celontin, for Petit mal, pyschomotor.

In three separate studies involving a total of 74 patients whose psychomotor seizures had been inadequately controlled by different drugs, Celontin was added. Sixty of the 74 patients showed improvement by a marked reduction in seizures. The criterion for improvement was reduction in seizure frequency of more than 50 per cent over the average for the previous three months while the patients were on other medication.

Celontin also demonstrated its value in the treatment of petit mal epilepsy seizures, according to separate studies of 126 patients. In petit mal seizures, which may be overlooked because frequently they last only a few seconds, the patient's eyelids or eyebrows twitch. He may stare vacantly and act slightly confused for a short period following the seizure.

In a published report on studies involving 54 patients, a high proportion of whom had intractable seizures, Celontin exhibited good anticonvulsant activity. In similar studies with 72 patients, it was concluded—on the basis of a high standard of seizure reduction—that Celontin is a drug of proved usefulness for petit mal.

Lift Truck LP-Gas Field Conversion Kit.—The Yale & Towne Manufacturing Company of Philadelphia has announced the development of the first industrial lift truck LP-Gas conversion kit which bears Underwriters' Laboratories listing for "approval by report" of a field installation.

This significant advancement in the use of liquefied petroleum gas as an industrial truck fuel was the result of a cooperative effort between the UL laboratories and the Yale engineering department. Its development means that the majority of Yale gasoline powered trucks currently in use may now be converted to LP-Gas by the user with a procedure and component parts which meet the stringent UL requirements for safety in operation.

In addition, Yale's introduction of this kit has provided local authorities having jurisdiction over LP-Gas industrial truck installations with a much needed yardstick against which to judge the safety of the application. The lack of this established standard for safe usage has been a serious deterrent to field conversion to LP-Gas and has also resulted in many poor and unsafe installations being put into service by lift truck users.

Descriptive material concerning the new Yale conversion kit, its parts and the procedure for its installation, is being distributed by Underwriters' Laboratories to interested local authorities—insurance inspectors, civil fire prevention officials and the like—so that they can properly evaluate the use of a truck converted to LP-Gas after determining the degree of fire hazard under which it will be working.

Each Yale conversion kit supplied to a user bears a serial number which gives certification that the conversion, if applied according to proper procedures, has been accomplished in a manner which meets UL specifications.

The basic components of the Underwriters' Listed Yale Kit are a fuel tank, a fuel filter, LP-Gas converter-regulator, carburetor, solenoid safety shut-off valve, static pressure relief valve and the hoses and connections necessary for installation.

The development of the UL listed conversion kit is Yale's second recent move toward assuring safer operation of industrial lift trucks. Last year the company introduced industry's first GS approved gasoline powered lift truck for use in certain locations classified as hazardous because of the threat of fire.

"Radio Pill" Developed for Medical Research Work.—A "radio pill" that sends out FM signals to medical researchers as it passes through the human body was demonstrated for the first time recently at the Rockefeller Institute.

Designed for research in the intestinal tract, the new "pill" is a plastic capsule one and one-eighth inches long and four-tenths of an inch in diameter. It is the world's smallest FM radio broadcast station.

The "radio pill" has been developed and tested jointly by the Rockefeller Institute, the New York Veterans Administration Hospital, and the Radio Corporation of America. It was designed by Dr. V. K. Zworykin, Honorary Vice President of RCA and Affiliate in Biophysics in the Medical Electronics Center of the Rockefeller Institute, and his associates, as it had been envisioned by Dr. John T. Farrar, Chief of the Gastroenterology Section of the New York Veterans Administration Hospital and Assistant Professor of Clinical Medicine at the Cornell University College of Medicine. The "pill" was developed by engineers of RCA's Commercial Electronics Products unit in Camden, N. J.

"The 'radio pill' seems to offer many possibilities as an important new tool in medical research," said Dr. Farrar. "It can be swallowed like any other medicinal capsule without discomfort, and will permit measurements on internal organs with minimum psychological and physical disturbance to normal bodily functions. It is hoped that the pill will prove valuable in studying human digestion and absorption in normal and pathological states. The new information which may be obtained on the physiology of muscular contractions is expected to be important in understanding gastro-intestinal disorders.

"The knowledge which is gained about the muscular activity of the right side of the colon, heretofore almost inaccessible to study, may prove useful in understanding the pathological physiology of such ailments as spastic colitis, ulcerative colitis and other organic and functional disease states. Besides measuring pressure changes in the digestive organs of the body, the 'radio pill' is being modified so that it may generate and transmit impulses relating to temperature within the gastro-intestinal tract."

In addition to Drs. Farrar and Zworykin, other officials at the demonstration were Dr. Detlev W. Bronk, President of the Rockefeller Institute; Dr. John B. Barnwell, Assistant Chief Medical Director for Research and Education of the Veterans Administration; and Dr. George H. Brown, Chief Engineer of RCA's Commercial Electronic Products unit.

Dr. Zworykin, who pioneered in the development of the television tube and also did fundamental work in perfecting the electron microscope, said the new "radio pill" has several electronic components. It consists of a tiny transistor, an oscillator, a ferrite cup inductance core and other circuit elements, and a minute, replaceable storage battery which powers the oscillator and has a life of fifteen hours. This battery is similar to the one used in the famous proximity fuse for anti-aircraft shells during World War II.

Heart of the capsule is the oscillator which is so sensitive that its frequency varies with changes in the pressure to which the "pill" is exposed. Information about these pressure changes is transmitted continuously in the form of FM radio signals that carry for a distance of several feet. These signals can be picked up on an outside FM radio receiver when an antenna is held close to the body. They can be recorded on one or more of three instruments: a meter, a recording galvanometer which makes a permanent record of wavy lines on paper much like an electrocardiograph, and a cathode ray oscillograph similar in principle to the picture tube of a home television set.

When the "pill" is swallowed by the patient, its course through the gastro-intestinal tract can be traced by fluoroscopy or other means. Since it has magnetic properties, it can be manipulated by magnetic forces outside the body. The capsule can be recovered and re-used in later experiments.

Studies involving use of the "radio pill" are carried out in the New York Veterans Hospital, for the time being at least, with the patient under continuous observation. In its present stage, the "pill" is an experimental technique. Its commercial possibilities will be evaluated following further extensive laboratory tests and experiments.

Working with Drs. Zworykin and Farrar are Carl Berkeley, Visiting Investigator at the Rockefeller Institute from RCA, and Fred L. Hatke, Rockefeller Institute electronics engineer. Both are members of the Medical Electronics Center at the Institute. Developments at RCA in Camden, New Jersey, have been under the direction of H. E. Haynes and A. L. Witchey. Preliminary testing has been conducted during the past year.